CLAS FINAL REVIEW

1. ABC Inc. produces widgets using two inputs. ABC's production function is $F(x_1, x_2) = 10x_1^{1/2} + 300x_2$, where x_1 is the number of units of input one per week and x_2 is the number of units of input two per week. Input one costs \$4 per unit and input two costs \$1,500 per unit. ABC Inc. can sell its widgets for \$4 per unit.

   a. Suppose that ABC Inc. has already purchased 10 units of input two. How many units of input one will it choose in order to maximize profits? What will its profits be?

☑ set up π function

$\pi = P[F(x_1, \bar{x}_2)] - w_1 x_1 - w_2 \bar{x}_2$

$= 4(10x_1^{1/2} + (300 \times 10)) - 4x_1 - (1500 \times 10)$

$= 40x_1^{1/2} + 12{,}000 - 4x_1 - 15{,}000$

$\boxed{\pi = 40x_1^{1/2} - 4x_1 - 3{,}000}$

→ observe relationship between x_1 and π

$x_1 \uparrow \rightarrow \pi \uparrow$ or $\downarrow$ [Ambiguity ⇒ use FOC]

☑ First order condition

$\frac{\partial \pi}{\partial x_1} = 20x_1^{-1/2} - 4 = 0$

$\frac{20}{x_1^{1/2}} = 4 \rightarrow 4x_1^{1/2} = 20 \rightarrow (x_1^{1/2} = 5)^2$

$\boxed{x_1^{SR} = 25}$

☑ second order condition

$\frac{\partial \pi^2}{\partial^2 x_1} = -10x_1^{-3/2} \rightarrow -\frac{10}{x_1^{3/2}} \rightarrow \frac{-10}{25^{3/2}} < 0$ π max, not min ✓

☑ shutdown condition

operate if π > − Fixed Cost

$\pi = 40(25)^{1/2} - 4(25) - 3{,}000 = -2900 > -\bar{F}\bar{K} = -15{,}000$ OPERATE

   b. Suppose that ABC Inc. can now choose both input one and input two. How many units of each input will it choose in order to maximize profits? What will its profits be?

☑ set up π function

$\pi = P[F(x_1, x_2)] - w_1 x_1 - w_2 x_2$

$= 4(10x_1^{1/2} + 300x_2) - 4x_1 - 1500x_2$

$\boxed{\pi = 40x_1^{1/2} - 4x_1 - 300x_2}$

→ observe interconnectedness

• NOT interconnected → separate π functions

$\pi_{x_1} = 40x_1^{1/2} - 4x_1$ $\pi_{x_2} = -300x_2$

$x_1 \uparrow \rightarrow \pi_{x_1} \uparrow$ or $\downarrow$ [Ambiguity ⇒ use FOC]

$x_2 \uparrow \rightarrow \pi_{x_2} \downarrow$

$\boxed{x_2^{LR} = 0}$

→ observe relationship between inputs and π

☑ First order condition

$\frac{\partial \pi_{x_1}}{\partial x_1} = 20x_1^{-1/2} - 4 = 0$

$\boxed{x_1^{LR} = 25}$

☑ second order condition

$\frac{\partial \pi_{x_1}^2}{\partial^2 x_1} = -10x_1^{-3/2} \rightarrow -\frac{10}{25^{3/2}} < 0$ π max, not min ✓

☑ shutdown condition

operate if π > 0

$\pi = 40(25)^{1/2} - 4(25) - 300(0) = 100$

100 > 0 → OPERATE

CLAS FINAL REVIEW

2. 123 LLP produces sculptures using labor and capital. 123 LLP's production function is F(L, K) = 100L + 2000K, where L is the hours of labor used per week and K is the number of 3D printers used per week. There are a limited number of workers with the skillset to produce 123 LLP's sculptures, therefore 123 LLP can use no more than 1,000 hours of labor per week. In addition, there are a limited number of 3D printers available; 123 LLP can use no more than ten 3D printers per week. The wage is $30 per hour and each 3D printer costs $500. 123 LLP can sell each sculpture for $1,000.
   a. Suppose that 123 LLP must produce 12,000 sculptures this week and it has already purchased eight 3D printers. What is the minimum possible total cost to produce 12,000 sculptures?

$Q = 100L + 2000\bar{K}$

$12{,}000 = 100L + (2000 \times 8)$

$12{,}000 = 100L + 16{,}000$

$100L = -4000$

$L = -40 \xrightarrow{\text{round up to } 0} L_Q^{SR} = 0$

$STC^{min} = wL_Q^{SR} + r\bar{K}$

$= (30 \times 0) + (500 \times 8)$

$\boxed{STC^{min} = \$4000}$

   b. Suppose that 123 LLP must produce 100,000 sculptures this week and it can choose both the hours of labor and the number of 3D printers. What is the minimum possible total cost to produce 100,000 sculptures?

$Q = 100L + 2000K$

perf subs → compare TRS to $\frac{w}{r}$

$TRS = \frac{100}{2000} = \frac{25}{500} \qquad \frac{w}{r} = \frac{30}{500}$

$TRS < \frac{w}{r}$ → use all K

$Q = 100L + 2000K$

$100{,}000 = (100 \times 0) + 2000K$

$K = 50$

$K_Q^{LR} = 10$

$Q = 100L + 2000K$

$100{,}000 = 100L + (2000 \times 10)$

$100{,}000 = 100L + 20{,}000$

$100L = 80{,}000$

$L_Q^{LR} = 800$ $[L_Q^{LR} < 1000 ✓]$

$LTC^{min} = wL_Q^{LR} + rK_Q^{LR}$

$= (30 \times 800) + (500 \times 10)$

$= 24{,}000 + 5{,}000$

$\boxed{LTC^{min} = \$29{,}000}$

To use all K, the firm would need 50 units. However, they can only buy a maximum of 10 units. Round K down to 10 and produce the rest of the units using L.

c. Suppose that 123 LLP can choose to produce as many sculptures as it wants, but it has already purchased five 3D printers. How many hours of labor will it use and how many sculptures will it produce?

☑ set up π function

$\pi = P[F(L, \bar{K})] - wL - r\bar{K}$

$= 1000(100L + (2000 \times 5)) - 30L - (500 \times 5)$

$= 100000L + 10{,}000{,}000 - 30L - 2500$

$\boxed{\pi = 99{,}970L + 9{,}997{,}500}$

→ observe relationship between L and π

$L\uparrow \rightarrow \pi\uparrow$ [use as much L as possible]

$\boxed{L^{SR} = 1000}$

$Q^{SR} = 100L^{SR} + 2000\bar{K}$

$Q^{SR} = (100 \times 1000) + (2000 \times 5)$

$Q^{SR} = 100{,}000 + 10{,}000$

$\boxed{Q^{SR} = 110{,}000}$

d. Suppose that 123 LLP can choose to produce as many sculptures as it wants, and it has not yet purchased any 3D printers. How many hours of labor will it use, how many 3D printers will it use, and how many sculptures will it produce?

☑ set up π function

$\pi = P[F(L,K)] - wL - rK$

$= 1000(100L + 2000K) - 30L - 500K$

$= 100000L + 2000000K - 30L - 500K$

$\boxed{\pi = 99{,}970L + 1{,}999{,}500K}$

→ observe interconnectedness

• not interconnected → separate π function

$\pi_L = 99{,}970L$ $\quad\quad$ $\pi_K = 1{,}999{,}500K$

$L\uparrow \rightarrow \pi_L\uparrow$ $\quad\quad$ $K\uparrow \rightarrow \pi_K\uparrow$

[use as much L as possible] $\quad\quad$ [use as much K as possible]

→ observe relationship between inputs and π

$\boxed{L^{LR} = 1000}$ $\quad$ $\boxed{K^{LR} = 10}$

$Q^{LR} = 100L^{LR} + 2000K^{LR}$

$Q^{LR} = (100 \times 1000) + (2000 \times 10)$

$Q^{LR} = 100{,}000 + 20{,}000$

$\boxed{Q^{LR} = 120{,}000}$

3. Smith, Jones, and Simpson LLC is a law firm with 12 attorneys. Each individual attorney has an identical production function: $q = \frac{3}{4} L$, where L is the number of hours worked per week by that attorney and q is the number of billable hours per week for that attorney. The firm can charge $500 for each billable hour. The firm's only input to production is labor, but it must also have an office space in order to operate. The rent on the office space is $1,000 per week, regardless of hours worked and billable hours. Each attorney can work a maximum of 60 hours per week.
   a. Suppose that the firm decides to pay its attorneys a fixed weekly salary. Each attorney will be paid $5,000 per week and will be required to work exactly 40 hours per week. What are the firm's weekly profits under this arrangement?

$q = \frac{3}{4} L$ $\qquad$ $Q = 12q \rightarrow Q = 9L$

total firm output

hrs per week per attorney

☑ set up π function

$\pi = PQ - [(\text{weekly salary}) \times (\text{\# of workers})] - \text{weekly rent}$

$= 500(9L) - (5000 \times 12) - 1000$

$= 4500L - 60{,}000 - 1{,}000$

$L = 40$ [required to work 40 hrs]

$= (4500 \times 40) - 60{,}000 - 1{,}000$

$= 180{,}000 - 60{,}000 - 1{,}000$

$\boxed{\pi = \$119{,}000/\text{wk}}$

b. Suppose that the firm decides to instead compensate its attorneys through profit sharing. Each attorney will receive no weekly or hourly salary. Instead, each attorney will simply be paid 50% of the ~~profits~~ revenue they earn for the firm (i.e. if one attorney earns $500 in profits for the week and another earns $800 in profits for the week, the first attorney will be paid $250 and the second will be paid $400). Under this arrangement, how many hours will each attorney work per week and what will the firm's weekly profits be?

Individual Attorney

☑ set up π function

π = 50% of revenue earned

$= 0.5\,pq$

$= 0.5 \times 500 \times \frac{3}{4} L$

$\boxed{\pi = 250\left(\frac{3}{4}L\right)}$

→ observe relationship between L and π

$L\uparrow \rightarrow \pi\uparrow$ [use as much L as possible]

$L^{LR} = 60$

Each attorney works 60 hours per week

Each attorney's earnings will be $250 \times \frac{3}{4} \times 60 = \$11{,}250$ per week

The Firm

☑ set up π function

π = Revenue − 50% of revenue − weekly rent

$= pQ - 0.5pQ - 1{,}000$

$= 0.5pQ - 1{,}000$

$= 0.5(500)(9L^{LR}) - 1{,}000$

$L^{LR} = 60$

$= (0.5 \times 500 \times 9 \times 60) - 1{,}000$

$\boxed{\pi = \$134{,}000/\text{wk}}$

c. Which arrangement does the firm prefer? Which arrangement do the attorneys prefer? Assume that the firm wants to maximize total profits and the attorneys want to maximize their own earnings.

The firm

The firm prefers the profit-sharing arrangement. Their profits under profit-sharing (\$134,000/wk) are higher than their profits under the weekly salary (\$119,000/wk)

Individual Attorneys

The attorneys also prefer the profit-sharing arrangement. Their earnings under profit-sharing (\$11,250/wk) are higher than their earnings under the weekly salary (\$5000/wk). Note that their hourly earnings under profit-sharing ($\frac{\$11{,}250}{60} = \$187.50$) are also higher than their hourly earnings under the weekly salary ($\frac{\$5000}{40} = \$125$).

4. Firm A's marginal product of labor is 27 units and its output is 100 units when labor is 4 hours and capital is 2 units. Firm B's marginal product of labor is 6 units and its output is 40 units when labor is 4 hours and capital is 2 units. Which of the following must be true?

FIRM A

$MP_L(4,2) = 27$

$F(4,2) = 100$

$AP_L = \frac{100}{4} = 25 \quad AP_K = \frac{100}{2} = 50$

FIRM B

$MP_L(4,2) = 6$

$F(4,2) = 40$

$AP_L = \frac{40}{4} = 10 \quad AP_K = \frac{40}{2} = 20$

a. Firm A's average product of labor at (5, 2) is greater than Firm B's average product of labor at (5, 2)

FIRM A

$MP_L(4,2) > AP_L(4,2)$

MP_L will drag AP_L upward

so $AP_L(5,2) > AP_L(4,2)$

FIRM B

$MP_L(4,2) < AP_L(4,2)$

MP_L will drag AP_L downward so

$AP_L(4,2) > AP_L(5,2)$

$AP_L^A(5,2) > AP_L^A(4,2) > AP_L^B(4,2) > AP_L^B(5,2)$

$\Rightarrow AP_L^A(5,2)$ must be greater than $AP_L^B(5,2)$

b. Firm A will choose more hours of labor to maximize its profits than Firm B will

Not necessarily true - we don't know what will happen to MP_L after (4, 2)

c. Firm B's average product of labor at (5, 2) is less than its average product of capital at (5, 2)

MUST BE TRUE

$AP_L(5,2) < AP_K(5,2)$

$AP_L(5,2) = \frac{F(5,2)}{5} \qquad AP_K(5,2) = \frac{F(5,2)}{2}$

[same numerator, AP_L has a larger denominator]

d. Firm A's marginal product of labor at (5, 2) is greater than its marginal product of labor at (4, 2)

Not necessarily true - we don't know what will happen to MP_L after (4, 2); it might decrease

e. Multiple answers above are correct

5. An accounting firm uses a single input to production, hours of labor, to produce tax returns. Currently, its marginal cost is $400 and its average cost is $550. Suppose that the government increases the cost of business licenses for accounting firms by $1,000. The licenses are imposed as a flat-fee before and after the cost increase. Assume that all else is unchanged. Which of the following must be true as a result of the increase in the cost of the business license?
   a. ~~Both the marginal cost and the average cost will increase~~
   b. ~~The average cost will increase and the effect on the marginal cost is ambiguous~~
   c. ~~The marginal cost will increase and the effect on the average cost is ambiguous~~
   d. The average cost will increase and the marginal cost will remain the same
   e. ~~None of the above~~

$MC = 400$ $AC = 550$

TC = variable cost + business license

$$\boxed{MC = \frac{\partial TC}{\partial Q}}$$

→ unaffected by business license; when you take the derivative, the business license will disappear because it is unaffected by Q

$$\boxed{AC = \frac{TC}{Q}}$$

→ increases when business license increases because the increase in the business license increases TC