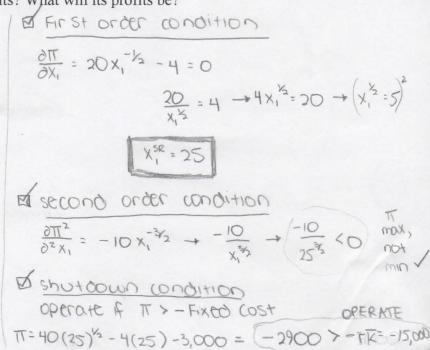
1. ABC Inc. produces widgets using two inputs. ABC's production function is  $F(x_1, x_2) = 10x_1^{1/2} + 300x_2$ , where  $x_1$  is the number of units of input one per week and  $x_2$  is the number of units of input two per week. Input one costs \$4 per unit and input two costs \$1,500 per unit. ABC Inc. can sell its widgets for \$4 per unit.

a. Suppose that ABC Inc. has already purchased 10 units of input two. How many units of input one

will it choose in order to maximize profits? What will its profits be?



b. Suppose that ABC Inc. can now choose both input one and input two. How many units of each input will it choose in order to maximize profits? What will its profits be?

If set up IT function

$$\pi = P[F(x_1, x_2)] - w_1 x_1 - w_2 x_2$$

$$= 4(10x_1^2 + 300x_2) - 4x_1 - 1500x_2$$

$$\pi = 40x_1^2 - 4x_1 - 300x_2$$

$$\Rightarrow observe interconnectedness$$
• NOT interconnected  $\Rightarrow$  separate
$$\pi \text{ functions}$$

$$\pi_{X_1} = 40x_1^2 - 4x_1, \qquad \pi_{X_2} = -300x_2$$

$$x_1 \uparrow \Rightarrow \pi_{X_1} \text{ or}$$

$$x_1 \uparrow \Rightarrow \pi_{X_1} \text{ or}$$

$$x_2 \uparrow \Rightarrow \pi_{X_2} \downarrow$$

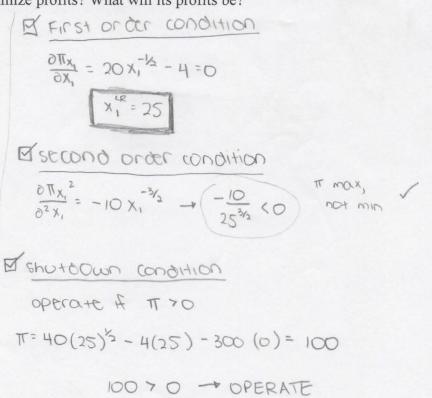
$$x_2 \uparrow \Rightarrow \pi_{X_2} \downarrow$$

$$x_2 \uparrow \Rightarrow \pi_{X_2} \downarrow$$

$$x_3 \uparrow \Rightarrow x_4 \downarrow$$

$$x_4 \uparrow \Rightarrow x_5 \downarrow$$

$$x_5 \downarrow \Rightarrow$$



- 2. 123 LLP produces sculptures using labor and capital. 123 LLP's production function is F(L, K) = 100L + 2000K, where L is the hours of labor used per week and K is the number of 3D printers used per week. There are a limited number of workers with the skillset to produce 123 LLP's sculptures, therefore 123 LLP can use no more than 1,000 hours of labor per week. In addition, there are a limited number of 3D printers available; 123 LLP can use no more than ten 3D printers per week. The wage is \$30 per hour and each 3D printer costs \$500. 123 LLP can sell each sculpture for \$1,000.
  - a. Suppose that 123 LLP must produce 12,000 sculptures this week and it has already purchased eight 3D printers. What is the minimum possible total cost to produce 12,000 sculptures?

$$Q = 100L + 2000K$$

$$12,000 = 100L + (2000 \times 8)$$

$$12,000 = 100L + 16,000$$

$$100L = -4000$$

$$L = -40 \frac{VP}{40} \qquad 1000$$

$$L = -40 \frac{VP}{40} \qquad 1000$$

b. Suppose that 123 LLP must produce 100,000 sculptures this week and it can choose both the hours of labor and the number of 3D printers. What is the minimum possible total cost to produce

100,000 sculptures?

Q = 100L + 2000K

PERF Subs 
$$\rightarrow$$
 compare

TRS to  $\frac{\omega}{\Gamma}$ 

TRS =  $\frac{100}{2000} = \frac{25}{500} = \frac{30}{\Gamma} = \frac{30}{500}$ 

TRS  $\leftarrow \frac{\omega}{\Gamma} = \frac{30}{500}$ 

TRS  $\leftarrow \frac{\omega}{\Gamma} = \frac{30}{500}$ 

K

Q =  $\frac{1000}{100} = \frac{1000}{1000} =$ 

$$|Q| = |OOL + 2000K$$

$$|OO,000| = |OOL + (2000 \times 10)$$

$$|OOL = 80,000$$

$$|LR = 800| [LR < 1000 \checkmark]$$

$$|LR = 800| + (500 \times 10)$$

$$= 24,000 + 5,000$$

$$|LR = 29,000|$$

TO use all k, the firm would need so units. However, they can only buy a maximum of 10 units. Round k down to 10 and produce the 10st of the units using L.

c. Suppose that 123 LLP can choose to produce as many sculptures as it wants, but it has already purchased five 3D printers. How many hours of labor will it use and how many sculptures will it

produce?

 $Q^{SR} = 1000$   $Q^{SR} = 1000 L^{SR} + 2000 K$   $Q^{SR} = (100 \times 1000) + (2000 \times 5)$   $Q^{SR} = 100,000 + 10,000$   $Q^{SR} = 110,000$ 

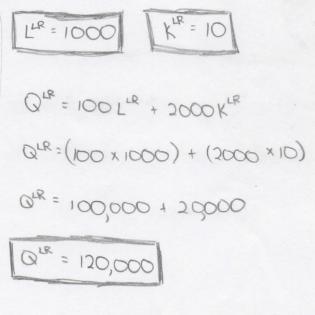
d. Suppose that 123 LLP can choose to produce as many sculptures as it wants, and it has not yet purchased any 3D printers. How many hours of labor will it use, how many 3D printers will it use, and how many sculptures will it produce?

DSET VE PELLING

DSET VE PELLING

DSECVE PEL

inputs and IT



- 3. Smith, Jones, and Simpson LLC is a law firm with 12 attorneys. Each individual attorney has an identical production function:  $q = \frac{3}{4} L$ , where L is the number of hours worked per week by that attorney and q is the number of billable hours per week for that attorney. The firm can charge \$500 for each billable hour. The firm's only input to production is labor, but it must also have an office space in order to operate. The rent on the office space is \$1,000 per week, regardless of hours worked and billable hours. Each attorney can work a maximum of 60 hours per week.
  - a. Suppose that the firm decides to pay its attorneys a fixed weekly salary. Each attorney will be paid \$5,000 per week and will be required to work exactly 40 hours per week. What are the firm's weekly profits under this arrangement?

$$Q = \frac{3}{4}L$$
 $Q = 12Q \rightarrow Q = 9L$ 

total

from

output

per attorner

b. Suppose that the firm decides to instead compensate its attorneys through profit sharing. Each attorney will receive no weekly or hourly salary. Instead, each attorney will simply be paid 50% of the profits they earn for the firm (i.e. if one attorney earns \$500 in profits for the week and another earns \$800 in profits for the week, the first attorney will be paid \$250 and the second will be paid \$400). Under this arrangement, how many hours will each attorney work per week and what will the firm's weekly profits be?

Individual Attorney

Set up IT function

IT = 50% of revenue earned

= 0.5 pg

= 0.5 × 500 × 3/4 L

IT = 250 (3/4 L)

→ abserve relationship between

L and IT

LY → IT \ [Use as much L]

as possible

Each attorney works
60 hours per week

PER WEEK

PER WEEK

PER WEEK

The Firm

Set up TT function

$$T = \text{Revenue} - 50\% \text{ of revenue} - \frac{\text{weeking}}{\text{rent}}$$
 $T = \text{Revenue} - 50\% \text{ of revenue} - \frac{\text{weeking}}{\text{rent}}$ 
 $T = \text{Revenue} - \frac{1000}{\text{rent}}$ 
 $T = 0.5 \text{ (500)} (9 \text{L}^{\text{LR}}) - 1,000$ 
 $T = 0.5 \text{ (500)} (9 \text{L}^{\text{R}}) - 1,000$ 
 $T = 0.5 \text{ (500)} (9 \text{L}^{\text{R}}) - 1,000$ 

c. Which arrangement does the firm prefer? Which arrangement do the attorneys prefer? Assume that the firm wants to maximize total profits and the attorneys want to maximize their own earnings.

The firm prefers the profit-sharing arrangement. Their profits under profit-sharing (\$134,000/wk) are higher than their profits under the weekly salary (\$119,000/wk)

# Individual Attorneys

The attorneys also prefer the profit-sharing arrangement.

Their earnings under profit-sharing (\$11,250/wk) are higher than their earnings under the weekly salarly (\$5000/wk).

Note that their hourly earnings under profit-sharing (\$11,250 : \$187.50) are also higher than their hourly earnings under profit sharing (\$5000 = \$125).

4. Firm A's marginal product of labor is 27 units and its output is 100 units when labor is 4 hours and capital is 2 units. Firm B's marginal product of labor is 6 units and its output is 40 units when labor is 4 hours and capital is 2 units. Which of the following must be true?

$$FIRM A$$
 $MP_L(4,2)=27$ 
 $F(4,2)=100$ 
 $AP_L = \frac{100}{4} = 25$ 
 $AP_K = \frac{100}{2} = 50$ 

FIRM B

$$MP_{L}(4,2)=6$$
 $F(4,2)=40$ 
 $AP_{L}=\frac{40}{4}=10$ 
 $AP_{K}=\frac{40}{2}=20$ 

a. Firm A's average product of labor at (5, 2) is greater than Firm B's average product of labor at (5,

FIRM B

MPL (4,2) > APL (4,2)

MPL will drag APL upward

MPL (4,2) > APL (4,2)

MPL (4,2) > APL (4,2)

MPL will drag APL

MPL (4,2) > APL (4,2)

MPL will drag APL

MPL (4,2) > APL (5,2) 
$$M$$

MPL will drag APL

MPL (4,2) > APL (5,2)  $M$ 

MPL (4,2) > APL (5,2)  $M$ 

MPL (4,2) > APL (5,2)  $M$ 

FIRM B

MP\_ 
$$(4,2)$$
 > AP\_  $(4,2)$  | AP\_  $(4,2)$  | AP\_  $(4,2)$  | AP\_  $(5,2)$  > AP\_  $(4,2)$  | AP\_  $(4,2)$  | AP\_  $(4,2)$  | AP\_  $(5,2)$  | AP\_  $(4,2)$  | AP\_  $(5,2)$  | AP\_  $(5$ 

b. Firm A will choose more hours of labor to maximize its profits than Firm B will

NOT necessarily true - we don't know what will happen to MP, after (4.2)

Firm B's average product of labor at 
$$(5, 2)$$
 is less than its average product of capital at  $(5, 2)$ 

AP<sub>L</sub>  $(5, 2) < AP_k (5, 2)$ 

$$AP_{L}(5,2) = F(5,2)$$

$$AP_{k}(s,2) = F(s,2)$$

$$AP_{k}(s,2) = F(s,2)$$

$$a larger denominator$$

d. Firm A's marginal product of labor at (5, 2) is greater than its marginal product of labor at (4, 2)

NOT DECESSORILY THUE - WE GOD'T KNOW WHAT WILL HAPPEN TO MPL after (4,2); it might decrease



e. Multiple answers above are correct

5. An accounting firm uses a single input to production, hours of labor, to produce tax returns. Currently, its marginal cost is \$400 and its average cost is \$550. Suppose that the government increases the cost of business licenses for accounting firms by \$1,000. The licenses are imposed as a flat-fee before and after the cost increase. Assume that all else is unchanged. Which of the following must be true as a result of the increase in the cost of the business license?

a. Both the marginal cost and the average cost will increase

b. The average cost will increase and the effect on the marginal cost is ambiguous

c. The marginal cost will increase and the effect on the average cost is ambiguous

d. The average cost will increase and the marginal cost will remain the same

None of the above

WC = 400

AC = 550

TC = variable cost + business license

MC = OTC
OQ

La unaffected by

business license;

when you take the

derivative, the business

license will disappear

because it is unaffected

by Q

 $AC = \frac{TC}{Q}$ 

the increases in the business
license increases because
the increase in the business
license in creases TC