**CLAS FINAL REVIEW**

1. I am writing a novel and am trying to decide how many pages to write. Under my contract, I will receive \$10 for every page I write. Because I have to take time off of work to write, I've determined that the cost of writing is $(p^2) / 100$ (the number of pages I write squared, divided by 100). In addition, if I choose to publish the book, I must pay my agent a flat-fee of \$3,000.
   a. *If I have already paid my agent's fee, how many pages should I write?*

FIRST: determine profit max # of pages

$\pi = \underbrace{\text{Revenue}}_{\text{(payment per page) × (# of pages "p")}} - \underbrace{\text{Total Costs}}_{\text{cost of writing + agent fee}}$

$\pi = 10p - \frac{p^2}{100} - 3000$

★ To maximize profits, derive profit with respect to p and set equal to 0

$\frac{\partial \pi}{\partial p} = 10 - \frac{p}{50} = 0$

$10 = \frac{p}{50}$

$p = 500$

$\frac{\partial^2 \pi}{\partial p^2} < 0$ ✓

If they choose to operate, they will write 500 pages; now, test if they will operate

Short run: operate if

profit if operate > profit if shut down

↓

$\pi$ if $p = 500$ > − Agent's fee

↓

$\pi = (10)(500) - \frac{500^2}{100} - 3000$

$= -500$

$-500 > -3000$ → OPERATE

$\boxed{p = 500}$

   b. If I have not yet paid my agent's fee, how many pages should I write?

$\pi = 10p - \frac{p^2}{100} - 3000$

$\frac{\partial \pi}{\partial p} = 10 - \frac{p}{50} = 0$

$10 = \frac{p}{50}$

$p = 500$

$\frac{\partial^2 \pi}{\partial p^2} < 0$ ✓

If they choose to operate, they will write 500 pages; now, test if they will operate

Long run: operate if

profit if operate > profit if shut down

↓

$\pi$ if $p = 500$ > 0

$-500 < 0$ → SHUT DOWN

$\boxed{p = 0}$

**CLAS FINAL REVIEW**

2. ABC Inc. is a customer service agency whose production is entirely based on the number of agents it hires. ABC's production function is $F(L) = 2L^{1/2}$, where L represents the total number of hours that ABC's agents work and F(L) represents the number of customer questions answered. When ABC hires agents it has two options: hire non-unionized agents or unionized agents. If ABC chooses to hire non-unionized agents, it must pay them $10 per hour. Because of the difficulty of finding non-unionized agents though, ABC will incur $50 in hiring costs if it chooses to hire non-unionized agents (the $50 is a flat cost that does not depend on the number of agents they ultimately hire; if they hire any non-unionized agents it costs them an extra $50). If ABC chooses to hire agents through the union, it must pay them $20 per hour. However, ABC will not incur any hiring costs if it hires agents through the union. Assume that ABC will either hire all non-unionized agents or all unionized agents, never a mix of both.

   a. Assume that ABC must answer 4 customer questions. How many hours will ABC's agents work? In addition, will ABC choose to hire non-unionized agents or unionized agents? What will the total cost of answering 4 customer questions be?

FIRST: determine how many hours are needed

$Q = 2L^{1/2}$

$L^{1/2} = \frac{Q}{2}$

$\boxed{L = \frac{Q^2}{4}} \rightarrow 4$

NEXT: determine whether to use unionized or non-unionized by comparing total costs

Non-unionized ✗

$TC = (\underbrace{\$10}_{w} \times \underbrace{4}_{L}) + \underbrace{\$50}_{\text{hiring costs}}$

$TC = \$90$

Unionized ✓

$TC = \underbrace{\$20}_{w} \times \underbrace{4}_{L}$

$\boxed{TC = \$80}$

   b. Assume that ABC can answer as many customer questions as it wants, so it will choose the amount that maximizes profits. Customers must pay $40 per question answered. How many hours will ABC's agents work? In addition, will ABC choose to hire non-unionized agents or unionized agents? How many customer questions will ABC answer?

Non-unionized

$\pi = P2L^{1/2} - wL - \text{hiring costs}$

$= 40(2L^{1/2}) - 10L - 50$

$\frac{\partial \pi}{\partial L} = \frac{40}{L^{1/2}} - 10 = 0$

$\frac{\partial^2 \pi}{\partial L^2} < 0$ ✓

$\boxed{L = 16}$

$\pi = 40(2\sqrt{16}) - (10)(16) - 50$

$\boxed{\pi = \$110}$

$F(16) = 2\sqrt{16} = \boxed{8 = Q}$

Unionized

$\pi = P2L^{1/2} - wL$

$= 40(2L^{1/2}) - 20L$

$\frac{\partial \pi}{\partial L} = \frac{40}{L^{1/2}} - 20 = 0$

$\frac{\partial^2 \pi}{\partial L^2} < 0$ ✓

$L = 4$

$\pi = 40(2\sqrt{4}) - (20)(4)$

$\pi = \$80$

$F(4) = 2\sqrt{4} = 4 = Q$

FINAL ANSWER: profit is higher with non-unionized → choose non-unionized

$\boxed{L = 16 \quad Q = 8}$

② b. Alternate method to find profit max # of customer questions (firm supply):

① $L_Q^* = \frac{Q^2}{4}$

② $TC = 10L + 50$

$= \frac{10Q^2}{4} + 50$

$= \frac{5Q^2}{2} + 50$

③ $MC = \frac{\partial TC}{\partial Q} = \frac{2(5Q)}{2} = 5Q$

④ $P = MC$

$40 = 5Q$

$\boxed{Q^* = 8}$

3. 123 Corp. makes blankets. 123 has two production inputs: workers and machines. 123 must pay $1 per worker and $100 per machine. There is a law in 123's city that says that any company with 101 or more employees must buy healthcare for all of its employees. If a company has 100 or less employees, it is not required to buy healthcare for any of its employees. 123 has determined that providing healthcare for its employees would cost $10,000, regardless of the number of employees covered. 123 is required under a contract with a customer to manufacture 200 blankets. Assume that 123 will not provide its employees with healthcare unless it is legally required to do so.
   a. 123 Corp.'s production function is F(L, K) = L + K, where L represents the total number of workers and K represents the total number of machines. Determine the most cost efficient way for 123 to produce 200 blankets. How much does each blanket cost on average?

MRTS = 1 $\quad \frac{w}{r} = \frac{1}{100}$ $\quad$ MRTS > $\frac{w}{r}$ → choose all L

★ comparing MRTS to $\frac{w}{r}$ does <u>not</u> factor in the cost of healthcare. Up until 100 workers, L is definitely better than K; after 100 workers, K may be better than L. To determine final answer, test total cost for two scenarios ① If all L (200, 0) ② If they stop using L right before they have to buy healthcare (100, 100)

Total cost ① (200, 0)

$Q = L + K$

$200 = L + 0$

$L_Q^{LR} = 200$

$LTC = wL_Q^{LR} + rK_Q^{LR} +$ healthcare (if $L \geq 101$)

$= \$1(200) + 0 + \$10{,}000$

$LTC = \$10{,}200$ ✗

Total cost ② $\boxed{(100, 100)}$

$Q = L + K$

$200 = \underbrace{100}_{\text{to avoid healthcare}} + K$

$K_Q^{LR} = 100$

$LTC = wL_Q^{LR} + rK_Q^{LR} +$ healthcare (if $L \geq 101$)

$= \$1(100) + \$100(100) + 0$

$\boxed{LTC = \$10{,}100}$ ✓

Option ② has lower total cost → $L_Q^{LR} = 100$ $K_Q^{LR} = 100$ $LTC = \$10{,}100$ $LAC = \$50.50$

   b. 123 Corp.'s production function is F(L, K) = $L^{1/2} K^{1/2}$, where L represents the total number of workers and K represents the total number of machines. Determine the most cost efficient way for 123 to produce 200 blankets. How much does each blanket cost on average?

FIRST: solve for demand functions

$Q = L^{1/2} K^{1/2}$

① $MRTS = \frac{K}{L}$

② $\frac{K}{L} = \frac{w}{r}$

③ $K = \frac{wL}{r}$

④ $Q = L^{1/2}\left(\frac{wL}{r}\right)^{1/2}$

$Q = \frac{L^{1/2} w^{1/2} L^{1/2}}{r^{1/2}}$

$Q = \frac{Lw^{1/2}}{r^{1/2}}$

⑤ $K = \frac{w}{r}(L)$

$K = \frac{w}{r}\left(\frac{Qr^{1/2}}{w^{1/2}}\right)$

$\boxed{L_Q^{LR} = \frac{Qr^{1/2}}{w^{1/2}}}$ → 2,000

$\boxed{K_Q^{LR} = \frac{Qw^{1/2}}{r^{1/2}}}$ → 20

*Like on part A, the normal cost min solving process does not account for the cost of healthcare. we must test to see if they would be better off using less than 101 workers and avoiding the healthcare expense

Total cost ① $\boxed{(2{,}000,\ 20)}$

$LTC = wL_Q^{LR} + rK_Q^{LR} +$ Healthcare (if $L \geq 101$)

$= \$1(2000) + \$100(20) + \$10{,}000$

$\boxed{LTC = \$14{,}000}$

Total cost ② $(100,\ 400)$

$Q = L^{1/2}K^{1/2}$

$200 = \underbrace{(100)^{1/2}}_{\text{to avoid healthcare}} K^{1/2}$

$200 = 10K^{1/2}$

$K_Q^{LR} = 400$

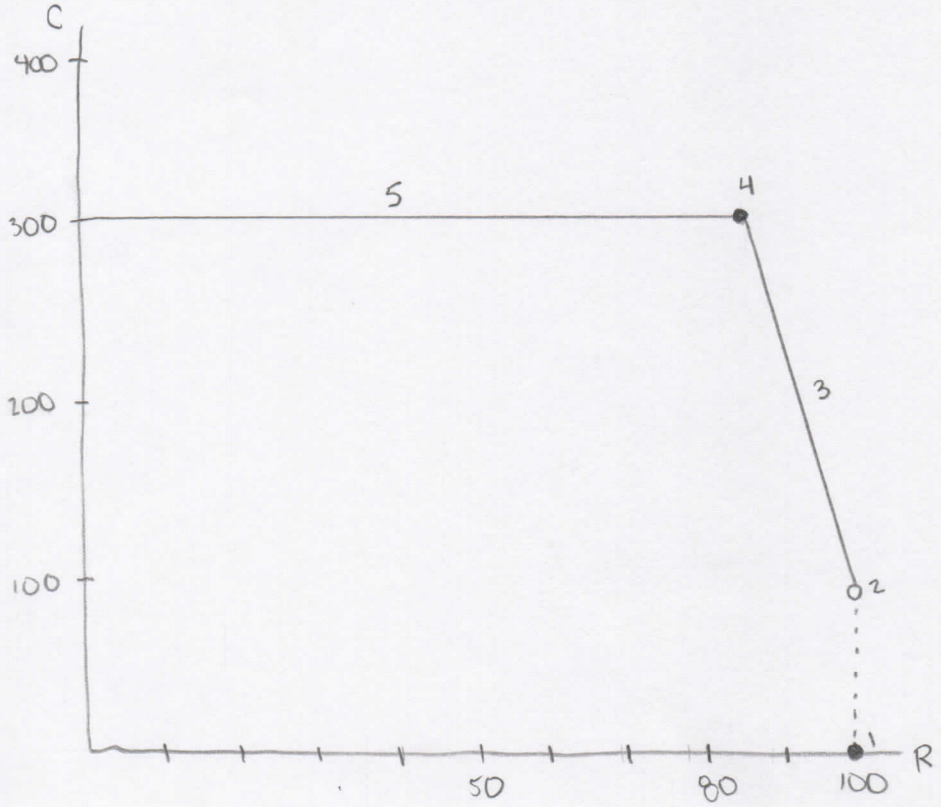
$LTC = wL_Q^{LR} + rK_Q^{LR} +$ Healthcare (if $L \geq 101$)

$= \$1(100) + \$100(400) + 0$

$LTC = \$40{,}100$

Option ① has lower total cost → $L_Q^{LR} = 2000$ $K_Q^{LR} = 20$ $LTC = \$14{,}000$ $LAC = \$70$

4. Justin is a sophomore in high school and he's trying to decide how many hours per week to work. Justin's parents want to encourage him to work, so they have told him that if he decides to get a job (even if he only works for a fraction of an hour per week) they will give him a weekly allowance of $100. If Justin does not work, his parents will give him no allowance money. If Justin decides to work, he will earn the minimum wage, $10/hour. Because Justin is only 16 years old he is not legally allowed to work more than 20 hours per week. The price of consumption is $1. Justin has no additional sources of income beyond money from working and allowance money.
   a. Draw and label Justin's budget constraint. Assume that Justin has 100 hours per week to allocate between recreation and labor.




1. If Justin does not work ($L=0$, $R=100$) his consumption will be $0. (100, 0) is an actual point on the B.C. so it is a closed point

2. Immediately after Justin begins working, his income rises by $100 (because of the money his parents give him). However (100, 100) is <u>not</u> an actual point on the B.C. so it is an open point

3. From (100, 0) to (80, 300) Justin is working. Thus, in the "working" portion of the B.C., consumption is increasing as Recreation decreases and Labor increases. Note that the slope on this portion is $\frac{w}{P} = \frac{\$10}{\$1} = 10$

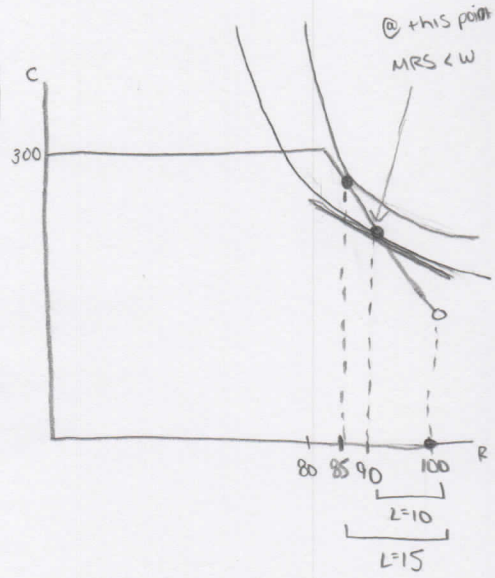
4. At ④, Justin has reached his maximum possible consumption. Labor is 20 hours per week (because Recreation is 80 hours per week), and Justin is not legally allowed to work more than 20 hours per week. At R = 80, L = 20 Justin's consumption is:

$$\underbrace{(\$10}_{\text{wage}} \times \underbrace{20)}_{\text{labor}} + \underbrace{\$100}_{\text{Allowance for working}} = \underbrace{\$300}_{\text{Total consumption}}$$

5. From (80, 300) to (0, 300) Justin is not working because that would exceed the legally permissible hours. Thus, in the "non-working" portion of the B.C., consumption remains flat because Justin is no longer earning additional money.

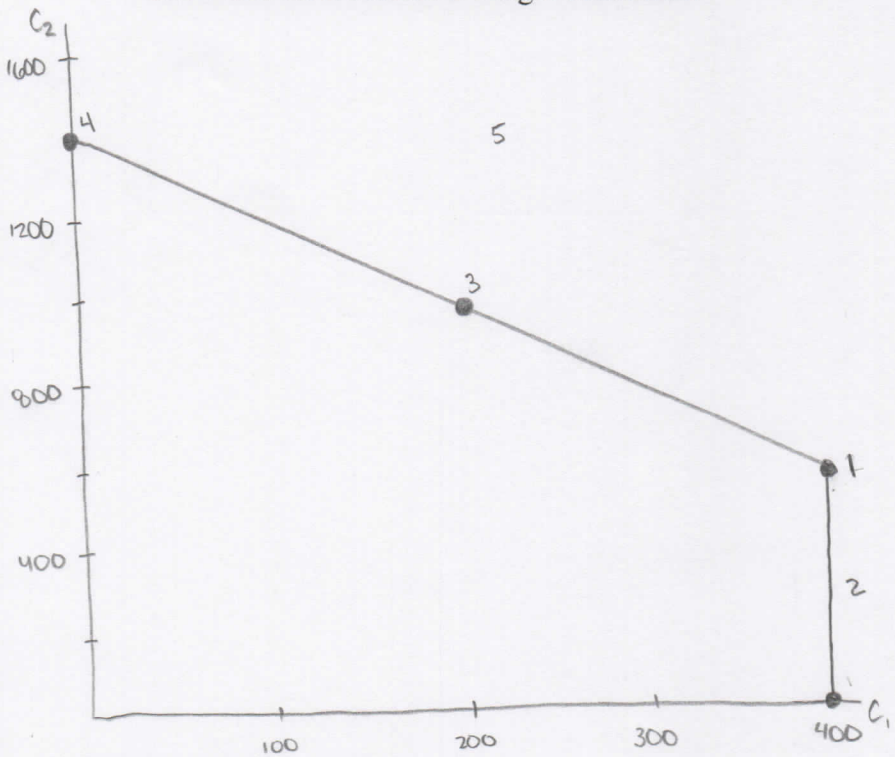
b. Assume that after ten hours of work the absolute value of Justin's MRS is 8. If Justin has a diminishing MRS, which of the following could be true?
   i. Justin's optimal amount of labor is 0 hours per week
   ii. Justin's optimal amount of labor is 5 hours per week
   iii. Justin's optimal amount of labor is 10 hours per week
   iv. Justin's optimal amount of labor is 15 hours per week
   v. Need more information

After ten hours of work, Justin is willing to work for \$8/hr. The actual wage is \$10/hr, so Justin will want to work more hours. He will work until his MRS equals the wage (as he works more hours, his MRS will continue increasing). Option iv is correct because it is the only option greater than 10 hours. Note that we can only pick iv because it says "could be true". If it said "must be true", iv would not be correct.




5. Jessica is trying to decide how to allocate her consumption between the present and the future. Jessica has split her life into two time periods: before she turns 25 (present) and after she turns 25 (future). Jessica will earn \$200 in the present and \$400 in the future. In addition, Jessica's grandfather has put \$600 in a trust fund for her. However, Jessica cannot access the trust fund money until she turns 25, so that money is only available for spending in the future. Assume the trust fund earns no interest

   a. Assume that there is no inflation and that the interest rate for both saving and borrowing is 100%. Draw and label Jessica's budget constraint.




1. (400, 600)

This is where Jessica reaches her maximum $C_1$. The calculation of $C_1$ is:

$$\underbrace{\$200}_{m_1} + \underbrace{\frac{\$400}{(1+1)}}_{\frac{m_2}{1+r}\ \text{(max loan)}} = \underbrace{\$400}_{C_1}$$

Note that the trust fund is not included in $C_1$ because it is not accessible in the present. Instead, it is included in $C_2$

2. This portion represents the trust fund of \$600. It is vertical because even if $c_2$ decreases, $c_1$ will not increase because the maximum possible $c_1$ has already been reached.

3. (200, 1000)

This is the endowment point, where Jessica is neither a saver nor a borrower. Jessica will spend everything in the exact period she earns it.

$$c_1 = m_1 = 200$$

$$c_2 = m_2 + \text{trust fund} = 1000$$

4. (0, 1400)

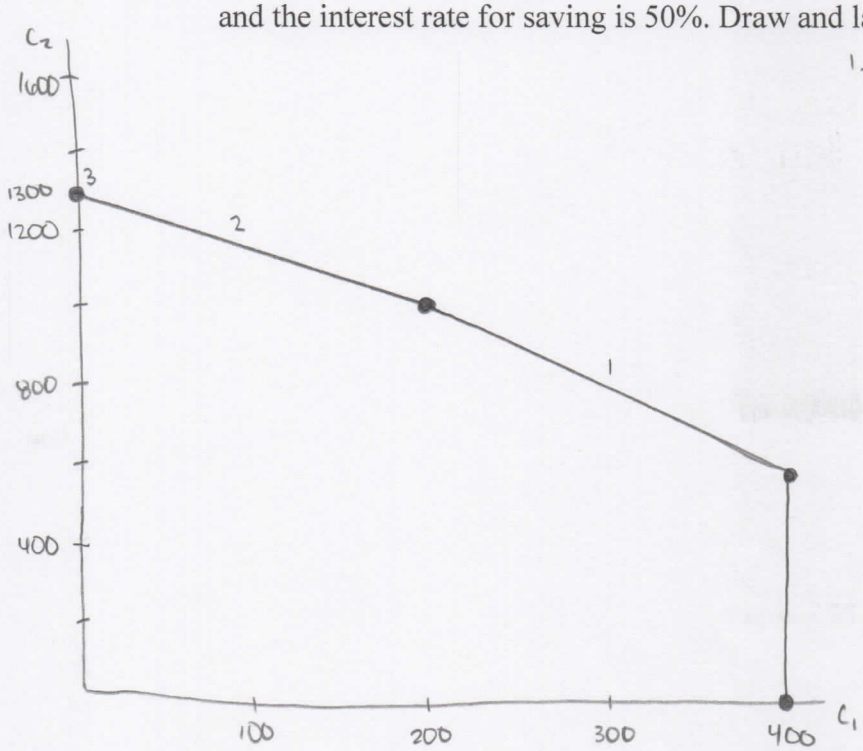
This is where Jessica reaches her maximum $c_2$. The calculation of $c_2$ is:

$$\underbrace{\$200 \times (1+1)}_{m_1 \times (1+r)} + \underbrace{\$400}_{m_2} + \underbrace{\$600}_{\text{Trust fund}} = \underbrace{\$1400}_{c_2}$$

5. Note that the slope from (0, 1400) to (400, 600) is

$$\frac{1+r}{1+\pi} = 2$$

b. Assume that there is no inflation. In addition, assume that the interest rate for borrowing is 100% and the interest rate for saving is 50%. Draw and label Jessica's budget constraint.



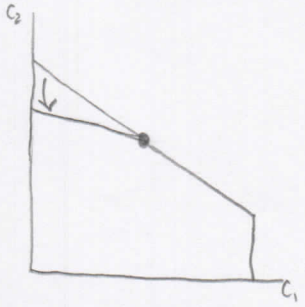

1. only the interest rate for savers changes, so all points in the borrower region remain the same. The slope from (200, 1000) to (400, 600) is $\frac{1 + r_{borrow}}{1 + \pi} = 2$

2. The slope from (0, 1300) to (200, 1000) is $\frac{1 + r_{save}}{1 + \pi} = \frac{3}{2}$

3. (0, 1300)
This is where Jessica reaches her maximum $C_2$

$$\underbrace{\$200 \times (1 + \tfrac{1}{2})}_{m_1 \times (1 + r_{save})} + \underbrace{\$400}_{m_2} + \underbrace{\$600}_{\text{must fund}} = \underbrace{\$1300}_{C_2}$$

c. Describe the substitution and income effects caused by the change in the interest rate for saving. Be sure to describe the effects in both of the following scenarios: (1) if Jessica was a saver before the change in the interest rate for saving and (2) if Jessica was a borrower before the change in the interest rate for saving.




① For savers the interest rate decreases. when the interest rate decreases, $C_1$ becomes relatively cheaper. so, SE on $C_1$ increases and SE on $C_2$ decreases. when the interest rate decreases, purchasing power decreases for savers (savers earn the interest rate, so when the interest rate decreases they earn less money) Purchasing power decreases and $C_1$ and $C_2$ are both normal, so the IE decreases for both.

| | $C_1$ | $C_2$ |
|---|---|---|
| SE | ↑ | ↓ |
| IE | ↓ | ↓ |

$r\downarrow$ PP.↓

② only the interest rate for savers changes, so for borrowers everything remains the same. Thus, borrowers will experience no SE or IE on $C_1$ or $C_2$. Intuitively, if you are a borrower and only the interest rate for savers changes, your behavior will be unaffected.

| | $C_1$ | $C_2$ |
|---|---|---|
| SE | 0 | 0 |
| IE | 0 | 0 |

6. Mike wants to invest in the stock market and he is trying to decide whether to make his own investment decisions or hire a broker. If Mike makes his own investment decisions, there is a 50% chance that his investments will be worth \$400 and a 50% chance that his investments will be worth \$900. The entire value of Mike's investments would be available for consumption. If Mike hires a broker, there is a 10% chance that his investments will be worth \$20,000 and a 90% chance that his investments will be worth \$200. However, the broker will charge Mike half of what his investments are worth, so only half the value of Mike's investments would be available for consumption. Mike's utility is given by $u(c) = c^{1/2}$, where c represents the amount of consumption. Determine whether it is better for Mike to make his own investment decisions or hire a broker. ★ Choose highest expected utility

Option ①: make own decisions

$E[u(C_a, C_b)] = \pi_a U_a + \pi_b U_b$

$\pi_a = \frac{1}{2}$ $\quad U_a = (400)^{1/2} = 20$

$\pi_b = \frac{1}{2}$ $\quad U_b = (900)^{1/2} = 30$

$E[u(C_a, C_b)] = \frac{1}{2}(20) + \frac{1}{2}(30)$

$= 25$

Option ②: broker

$20{,}000 \rightarrow C_a = 10{,}000$

$200 \rightarrow C_b = 100$

$E[u(C_a, C_b)] = \pi_a U_a + \pi_b U_b$

$\pi_a = \frac{1}{10}$ $\quad U_a = (10{,}000)^{1/2} = 100$

$\pi_b = \frac{9}{10}$ $\quad U_b = (100)^{1/2} = 10$

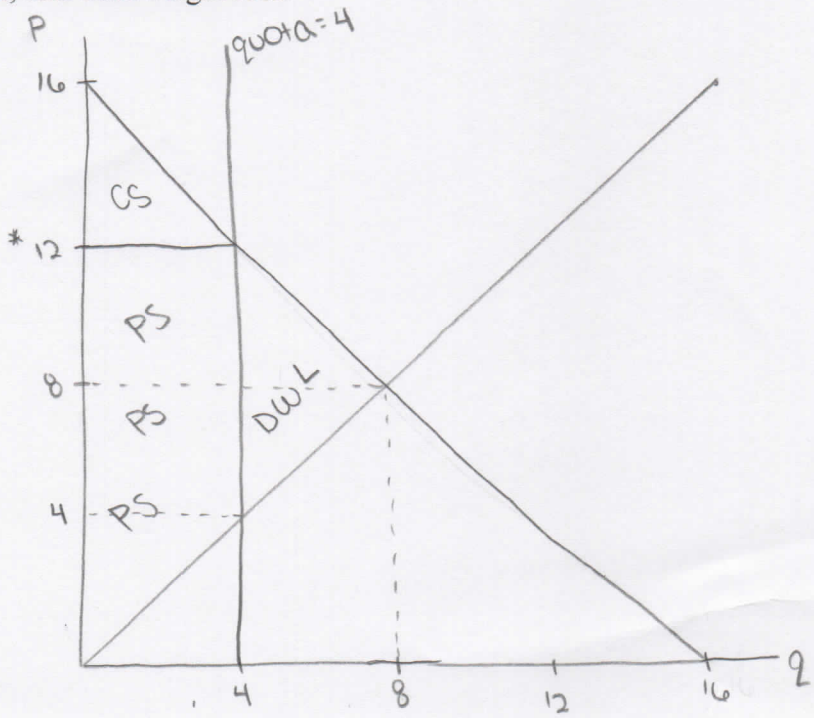
$E[u(C_a, C_b)] = \frac{1}{10}(100) + \frac{9}{10}(10)$

$= 19$

Higher expected utility if he makes his own decisions → **He will make his own decisions**

7. Inverse demand is given by p = 16 – q. Inverse supply is given by p = q. A new law is passed stating that it is illegal to sell more than 4 units of this product. Find the new price, consumer surplus, producer surplus, and deadweight loss.

*sellers will sell at the highest price possible → p = \$12




Equilibrium

$16 - q = q$ $\quad p = 16 - q$

$16 = 2q$ $\quad p = 16 - 8$

$q = 8$ $\quad p = 8$

Quota

$p = 16 - q$ $\quad p = q$

$p = 16 - 4$ $\quad p = 4$

$p = 12$

$CS = \frac{1}{2}(4 \times 4) = \$8$

$PS = (8 \times 4) + \frac{1}{2}(4 \times 4) = \$40$

$DWL = \frac{1}{2}(8 \times 4) = \$16$