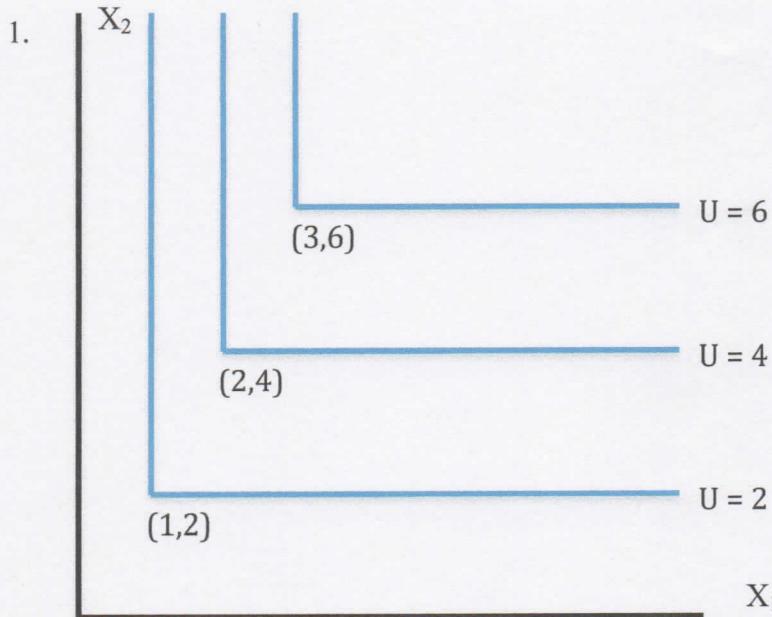


CLAS MIDTERM 1 REVIEW



L-shaped IDC's

→ perfect comps

→ utility function has "min"

- a. Write the utility function that corresponds to the graph

GENERAL FORMULA : $U(x_1, x_2) = A \min\left[\frac{x_1}{a}, \frac{x_2}{b}\right]$

\downarrow
 monotonic transformation
 \searrow
 kink point

$$U(2, 4) = 4$$

$$4 = A \min\left[\frac{2}{2}, \frac{4}{4}\right]$$

$$4 = A \min[1, 1]$$

$$4 = 1 \times A \rightarrow A = 4$$

USING
* $(2, 4)$

$$U(x_1, x_2) = \min\left[\frac{x_1}{2}, \frac{x_2}{4}\right]$$

$$U(x_1, x_2) = 4 \min\left[\frac{x_1}{2}, \frac{x_2}{4}\right] = \min[2x_1, x_2]$$

$$\textcircled{1} \quad U(x_1, x_2) = \min[2x_1, x_2]$$

$$2x_1 = x_2$$

$$\textcircled{3} \quad P_1 x_1 + P_2 (2x_1) = m$$

$$x_1 (P_1 + 2P_2) = m$$

$$\textcircled{2} \quad x_2 = 2x_1$$

$$x_2 = 2 \left(\frac{m}{P_1 + 2P_2} \right)$$

$$x_1 = \frac{m}{P_1 + 2P_2}$$

$$x_1^*(P_1, P_2, m) = \frac{m}{P_1 + 2P_2}$$

$$x_2^*(P_1, P_2, m) = \frac{2m}{P_1 + 2P_2}$$

* TRY USING (1, 2) or (3, 6)
instead — you will get
to the same final answer!

CLAS MIDTERM 1 REVIEW

c. Prove demand for x_1 is scale invariant (homogenous of degree zero)

$$\text{If scale invariant: } x_1^*(p_1, p_2, m) = x_1^*(\lambda p_1, \lambda p_2, \lambda m)$$

$$x_1^*(\lambda p_1, \lambda p_2, \lambda m) = \frac{(\lambda m)}{(\lambda p_1) + 2(\lambda p_2)} = \frac{\lambda m}{\lambda(p_1 + 2p_2)} = \frac{m}{p_1 + 2p_2} = x_1^*(p_1, p_2, m) \quad \checkmark$$

d. What is MU_1 at (2, 2)? What is MU_2 at (2, 2)? special method for perf comps:

$$MU = u(\text{end}) - u(\text{start})$$

$$MU_1 = u(3, 2) - u(2, 2)$$

Extra utility w/one more
of x_1 , keeping x_2 the same

$$\begin{aligned} MU_1 &= \min[(2 \times 3), 2] - \min[(2 \times 2), 2] \\ &= \min[6, 2] - \min[4, 2] \\ &= 2 - 2 \end{aligned}$$

$$MU_1 = 0$$

$$2. \quad U(x_1, x_2) = (1/3)x_1 + (1/4)x_2 \quad p_2 = (3/4)p_1 \quad m = 10p_1$$

Solve for demands for x_1 and x_2

$$MRS = \frac{MU_1}{MU_2} = \frac{\left(\frac{1}{3}\right)}{\left(\frac{1}{4}\right)} = \frac{4}{3}$$

$$P_2 = \frac{3P_1}{4}$$

$$3P_1 = 4P_2$$

$$\frac{P_1}{P_2} = \frac{4}{3}$$

$$MRS = \frac{P_1}{P_2}$$



The consumer is indifferent between all affordable bundles. Note that the minimum possible demands for x_1 and x_2 are 0, and the maximum possible demands are $x_1^* = \frac{m}{P_1} = \frac{10P_1}{P_1} = 10$ and $x_2^* = \frac{m}{P_2} = \frac{10P_1}{\left(\frac{3P_1}{4}\right)} = \frac{40}{3}$

CLAS MIDTERM 1 REVIEW

3. $U(A, B) = A + \ln(B)$, where A is the number of apples and B is the number of bananas.

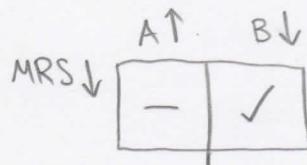
- a. Solve for the demands for apples and bananas. Assume the price of apples is \$10, the price of bananas is \$1 and income is \$5.

$$\textcircled{1} U(A, B) = A + \ln B$$

$$P_A A + P_B B = M$$

$$\textcircled{2} MRS = \frac{MU_A}{MU_B} = \frac{1}{\left(\frac{1}{B}\right)} = B$$

$$\textcircled{4} P_A A + P_B \left(\frac{P_A}{P_B}\right) = M$$



$D M R S \rightarrow \text{convex} \rightarrow$ USE tangency

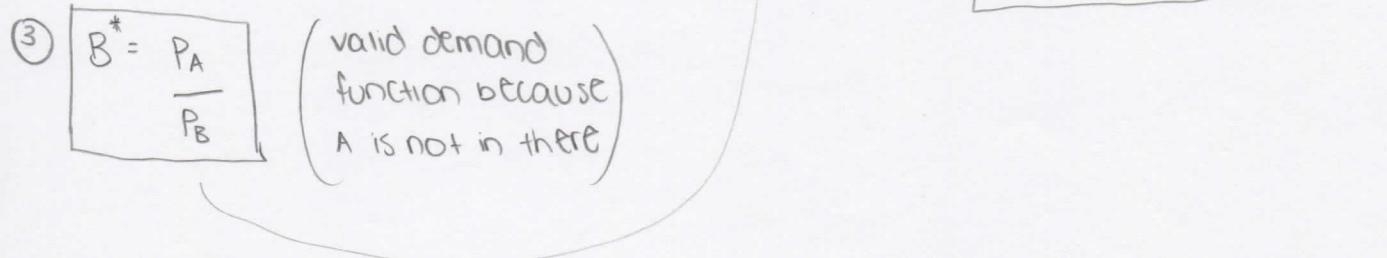
$$P_A A + P_B B = M$$

$$P_A A = M - P_B B$$

$$A^* = \frac{M - P_B B}{P_A}$$

$$\textcircled{3} B^* = \frac{P_A}{P_B}$$

(valid demand function because A is not in there)



$$\textcircled{5} A^* = \frac{m - P_A}{P_A} = \frac{5 - 10}{10} = -\frac{5}{10} = -\frac{1}{2}$$

can't have negative demand; must round up to 0

$$A^* = 0$$

$$B^* = \frac{P_A}{P_B} = \frac{10}{1} = 10$$

changing A^* throws the budget constraint out of balance; must recalculate B^* using budget constraint

$$B^* = 5$$

$$P_A A + P_B B = M$$

$$(10 \times 0) + (1 \times B) = 5$$

$$B = 5$$

CLAS MIDTERM 1 REVIEW

- b. Solve for the demands for apples and bananas. Assume that apples are always free and bananas cost \$1 each. Income is \$5. Assume that consumers are allowed to purchase a maximum of 10 apples and a maximum of 10 bananas.

The consumer always likes both apples and bananas ($A \uparrow \rightarrow u \uparrow$ and $B \uparrow \rightarrow u \uparrow$). So, if cost was not a factor, they would consume as much as possible of A and B. Apples are free, so they will consume the maximum possible amount: 10. They will then spend all of their income on bananas; they can afford $\frac{m}{P_b} = \frac{\$5}{\$1} = 5$ units.

$$\boxed{A^* = 10}$$
$$B^* = 5$$

- c. Is MU_A diminishing? Is MU_B diminishing?

$$MU_A = 1$$

$$A \uparrow \Rightarrow MU_A -$$

MU_A is constant, not diminishing

$$MU_B = \frac{1}{B}$$

$$B \uparrow \Rightarrow MU_B \downarrow$$

MU_B is diminishing

CLAS MIDTERM 1 REVIEW

4. John's utility is given by, $u(b, c) = bc - 1000$, where b is the number of books and c is the number of cars.

Assume that the price of books is \$10 each and the price of cars is \$20,000 each. John has \$100,000 to spend.

- a. Find utility at the point (10, 10)

$$\begin{aligned} u(10, 10) &= (10 \times 10) - 1000 \\ &= 100 - 1000 \end{aligned}$$

$$u(10, 10) = -900$$

Note that utility does not mean that x and y are goods. Utility is negative because of a monotonic transformation. As b increases, utility increases and as c increases, utility increases, so they are goods

- b. Solve for John's demands for books and cars. Assume John *cannot* consume fractions of books or cars.

$$\textcircled{1} \quad u(b, c) = bc - 1000$$

$$\textcircled{5} \quad p_b b + p_c c = m$$

$$\textcircled{6} \quad c = \frac{p_b b}{p_c}$$

$$\textcircled{2} \quad MRS = \frac{MU_b}{MU_c} = \frac{c}{b}$$

$$p_b b + p_c \left(\frac{p_b b}{p_c} \right) = m$$

$$= \frac{p_b}{p_c} (b)$$

$$\textcircled{3} \quad \frac{c}{b} = \frac{p_b}{p_c}$$

$$p_b b + p_c b = m$$

$$= \frac{p_b}{p_c} \left(\frac{m}{2p_b} \right)$$

$$\textcircled{4} \quad c = \frac{p_b b}{p_c}$$

$$2p_b b = m$$

$$b = \frac{m}{2p_b}$$

$$c = \frac{m}{2p_c}$$

$$\boxed{b^*(p_b, p_c, m) = \frac{m}{2p_b}}$$

$$\boxed{c^*(p_b, p_c, m) = \frac{m}{2p_c}}$$

$$\textcircled{1} \quad b^*(p_b, p_c, m) = \frac{m}{2p_b} = \frac{100,000}{2(10)} = 5000$$

$$c^*(p_b, p_c, m) = \frac{m}{2p_c} = \frac{100,000}{2(20,000)} = 2.5$$

→ can't have fractions; two options: round up or round down. Test utility for both to see which is better

Rounding down

$$(C^* = 2)$$

$$p_b b + p_c c = m$$

$$(10b) + (20000 \times 2) = 100,000$$

$$10b = 60,000$$

$$b^* = 6000$$

$$u(6000, 2) = (6000 \times 2) - 1000 = \underline{\underline{11,000}}$$

Rounding up

$$(C^* = 3)$$

$$p_b b + p_c c = m$$

$$10b + (20000 \times 3) = 100,000$$

$$10b = 40,000$$

$$b^* = 4000$$

$$u(4000, 3) = (4000 \times 3) - 1000 = \underline{\underline{11,000}}$$

FINAL ANSWER:

indifferent between
(2, 6000) and (3, 4000)

CLAS MIDTERM 1 REVIEW

5. $U(x_1, x_2) = x_1 + x_2^2$ $m = 20, p_1 = \$2, p_2 = \10

Solve for the demand functions for x_1 and x_2

$$① U(x_1, x_2) = x_1 + x_2^2$$

$$② MRS = \frac{MU_1}{MU_2} = \frac{1}{2x_2}$$

$x_1 \uparrow$	$x_2 \downarrow$
MRS ↓	- X

MRS not diminishing

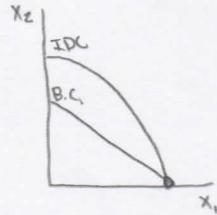
MRS is increasing
→ concave

DO not use tangency condition

If all x_1

$$\text{Afford } \frac{m}{P_1} = \frac{20}{2} = 10 \text{ units}$$

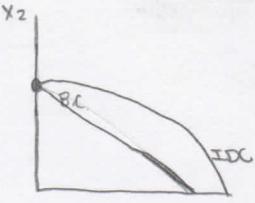
$$U(10,0) = 10 + 0^2 = 10$$



If all x_2

$$\text{Afford } \frac{m}{P_2} = \frac{20}{10} = 2 \text{ units}$$

$$U(0,2) = 0 + 2^2 = 4$$



$$U(10,0) > U(0,2) \rightarrow$$

$x_1^* = 10$
$x_2^* = 0$

CLAS MIDTERM 1 REVIEW

6. Assume there are only two goods that can be consumed: X and Y. Katie and Lauren both have Cobb-Douglas utility functions, and Lauren's utility function is a monotonic transformation of Katie's function. Based on these facts, determine whether each of the following statements (independently) are true or false and explain your answers in three sentences or less.

- Assuming that Katie and Lauren face the same prices and have the same income, it is possible that Katie's optimal consumption of X is higher than Lauren's optimal consumption of X.

Not possible

Utility functions that have a monotonic transformation share the same MRS. If they both have the same MRS, both have Cobb-Douglas utility, and face the same prices and income, they will have to have the same optimal consumption.

Ex:

$$\begin{aligned} \textcircled{1} \quad & U(x, y) = x^3 y^3 & P_x x + P_y y = m \\ \textcircled{2} \quad & MRS = \frac{y}{x} \checkmark & \textcircled{3} \quad P_x x + P_y \left(\frac{P_x x}{P_y} \right) = m \\ \textcircled{4} \quad & y = \frac{P_x x}{P_y} & 2P_x x = m \\ \textcircled{5} \quad & \boxed{y^* = \frac{m}{2P_y}} & \boxed{x^* = \frac{m}{2P_x}} \end{aligned}$$

$$\begin{aligned} \textcircled{1} \quad & U(x, y) = 8x^2 y^2 & \textcircled{2} \quad MRS = \frac{y}{x} \checkmark \\ \downarrow & & \\ \boxed{x^* = \frac{m}{2P_x}} & & \boxed{y^* = \frac{m}{2P_y}} \end{aligned}$$

- It is possible that Katie and Lauren's marginal utilities of good Y differ at the bundle (4, 6).

Possible

Monotonic transformations preserve the order of preferences, but they do not preserve total utility or marginal utility.

Ex:

$$U(x, y) = x^3 y^3$$

$$MU_y = 3x^3 y^2$$

$$MU_y(4, 6) = 3(4^3)(6^2)$$

$$= 6912$$

$$U(x, y) = x^2 y^2$$

$$MU_y = 2x^2 y$$

$$\begin{aligned} MU_y(4, 6) &= 2(4^2)(6) \\ &= 192 \end{aligned}$$