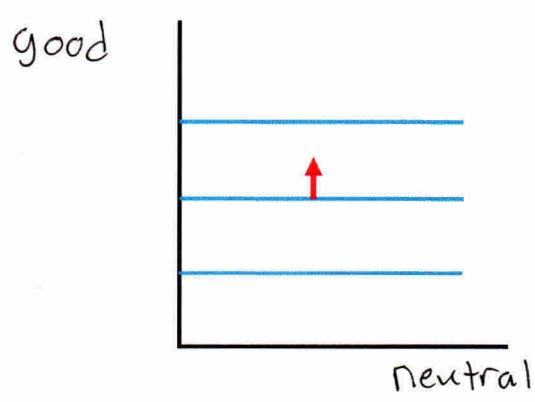
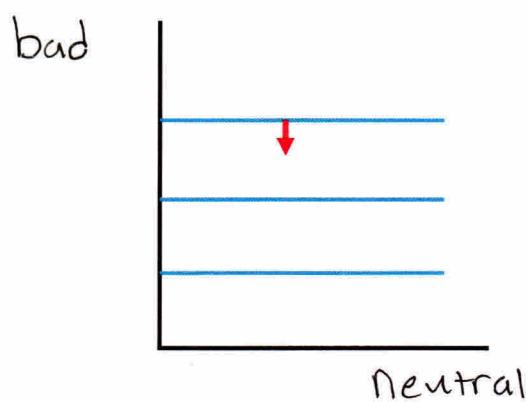
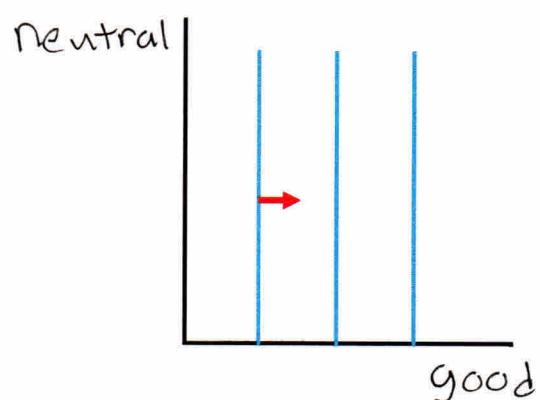
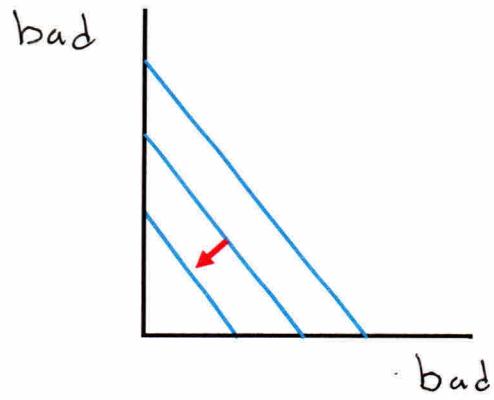


Label the X and Y axis for the following indifference curves (Good, Bad, Neutral)



$$4. U(X, Y) = X + Y^{1/2}/2 \leftarrow \begin{aligned} M_{UX} &= 1 \\ M_{UY} &= \frac{Y^{-\frac{1}{2}}}{4} \end{aligned} \leftarrow \begin{array}{l} P_x = 8 \quad P_y = 1 \quad M = 20 \\ \text{Quasi-linear} \\ \text{Convex} \end{array}$$

- a. Find the optimal consumption of X and Y.
- b. Prove that the demand function for Y from part (a) is a true demand function
- c. Is Y normal or inferior? Is Y ordinary or Giffen? Is Y a substitute or complement?

a. Quasi-linear convex. use Tangency

$$MRS = \frac{1}{4Y^{-\frac{1}{2}}} = 4Y^{\frac{1}{2}} \rightarrow MRS = \frac{P_x}{P_y} \rightarrow 4Y^{\frac{1}{2}} = \frac{P_x}{P_y} \rightarrow Y^* = \frac{P_x^2}{16P_y^2}$$

Plug  $Y^*$  into BC

$$P_x X + P_y \left( \frac{P_x^2}{16P_y^2} \right) = M \rightarrow P_x X = M - \left( \frac{P_x^2}{16P_y} \right)$$

$\downarrow$

$X^* = \frac{M}{P_x} - \frac{P_x}{16P_y}$

$$X^* = \frac{20}{8} - \frac{8}{16} = 2$$

b. To prove we have a true demand function, we must prove that  $Y^*$  is scale invariant:  $Y^*(P_x, P_y, M) = Y^*(\lambda P_x, \lambda P_y, \lambda M)$

$$Y^* = \frac{P_x^2}{16P_y^2} \rightarrow Y^*(\lambda P_x, \lambda P_y, \lambda M) = \frac{(\lambda P_x)^2}{16(\lambda P_y)^2} = \frac{\lambda^2 P_x^2}{16\lambda^2 P_y^2} = \frac{P_x^2}{16P_y^2}$$

$$Y^*(P_x, P_y, M) = Y^*(\lambda P_x, \lambda P_y, \lambda M) \quad \checkmark$$

so we have a true demand function.

c.  $Y^* = \frac{P_x^2}{16P_y^2}$

since there is no M in our  $Y^*$  equation, Y is a zero income effect good.

since an increase in  $P_y$  decreases demand for Y, Y is ordinary.

since an increase in  $P_x$  increases demand for Y, Y is a substitute for X.

5. Taylor is trying to decide between consuming (g)inger beer and (a)lcoholic ginger beer. Each bottle of ginger beer increases Taylor's utility by 6 utils, and each bottle of alcoholic ginger beer increases Taylor's utility by 8 utils. Taylor's marginal utility of for both types of ginger beer is constant. Taylor has \$24 to spend, and the price of ginger beer is \$3 while the price of alcoholic ginger beer \$4. However, Taylor has 6 coupons for \$1 off of the price of ginger beer. Graph Taylor's budget constraint with ginger beer on the X axis. Be sure to label all important parts of the graph (intercepts, slopes, kink points, etc). How much ginger beer and alcoholic ginger beer does Taylor consume?

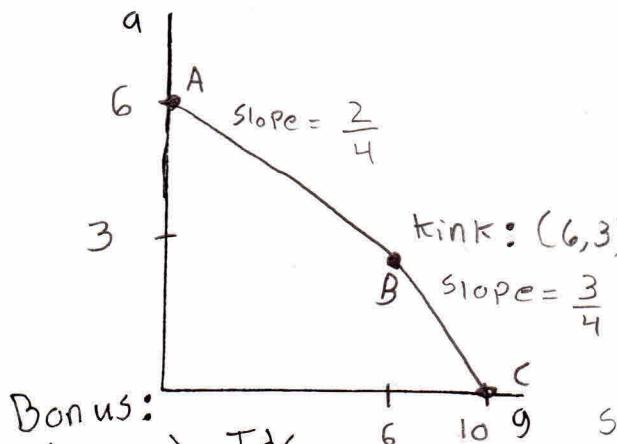
Since Taylor's  $M_{ug}$  and  $M_{ua}$  are both constant, we must be dealing with perfect subs. Using this information along with  $M_{ug} = 6$  and  $M_{ua} = 8$ , we can construct our utility function:

$$U(g, a) = 6g + 8a$$

Our BC information is as follows:  $P_g = 3$   $P_a = 4$   $M = 24$   
 $P_{gc} = 2$  with 6 coupons

Now we can construct our BC:

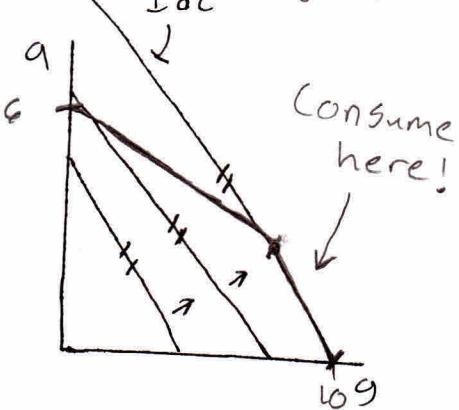
Compare Bang for Buck



$$\frac{M_{ug}}{P_g} \text{ vs } \frac{M_{ug}}{P_{gc}} \text{ vs } \frac{M_{ua}}{P_a}$$

$$\frac{6}{3}^{\text{2nd}} \text{ vs } \frac{6}{2}^{\text{1st}} \text{ vs } \frac{8}{4}^{\text{2nd}}$$

Coupons ginger beer is the best, so consume all couponed ginger beer. The bang for buck of ginger beer is equal to that of alcoholic ginger beer, so once we have used all coupons, we are indifferent between the two. Any point on the BC between B and C will maximize utility.



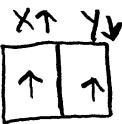
$$U(X, Y) = X^2 + Y^3$$

a. Solve for  $X^*$  and  $Y^*$

$$P_x = 4 \quad P_y = 8 \quad M = 24$$

Here we are given a non-typical utility function that does not match any of our 6 types. Test the MRS to learn more about the utility function.

- Dmrs  $\rightarrow$  convex  $\rightarrow$  can use tangency
- Imfs  $\rightarrow$  concave  $\rightarrow$  all of X or all of Y (boundary)
- Cmfs  $\rightarrow$  linear  $\rightarrow$  compare mfs to  $\frac{P_x}{P_y}$

•  $MRS = \frac{2X}{3Y} \rightarrow$   MRS  $\rightarrow$  Imfs

All X

$$X = \frac{M}{P_x} = \frac{24}{4} = 6$$

All Y

$$Y = \frac{M}{P_y} = \frac{24}{8} = 3$$
$$U(X, Y) = 6^2 + (0)^3 = \underline{\underline{36}} \qquad U(X, Y) = (0)^2 + (3)^3 = \underline{\underline{27}}$$

Consume all X

$$\boxed{X^* = 6 \quad Y^* = 0}$$

- a) Find the Marshallian, i.e. utility maximizing, demands for goods x and y for a consumer utility given by  $u(x, y) = \min\left\{\frac{1}{3}x + 1, y\right\}$ ,  $p_x = 6$ ,  $p_y = 3$ ,  $m = 24$ , and graph a few indifference curves and the given budget constraint

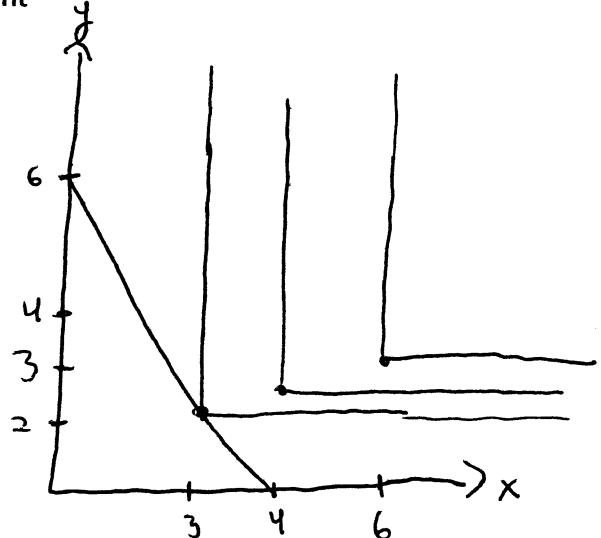
$$\frac{1}{3}x + 1 = y$$

$$\Rightarrow p_x x + p_y (\frac{1}{3}x + 1) = m$$

$$\Rightarrow p_x x + \frac{1}{3}p_y x + p_y = m$$

$$\Rightarrow \boxed{x^* = \frac{m - p_y}{p_x + \frac{1}{3}p_y}} = \frac{24 - 3}{6 + \frac{1}{3}(3)} = 3$$

$$\Rightarrow \boxed{y^* = \frac{1}{3}\left(\frac{m - p_y}{p_x + \frac{1}{3}p_y}\right) + 1} = \frac{m - p_y}{3p_x + p_y} + \frac{3p_x + p_y}{3p_x + p_y} \\ = \boxed{\frac{m + 3p_x}{3p_x + p_y}} = \frac{24 + 3(6)}{3(6) + 3} = 2$$



- b) Find the MRS at the points  $(6, 3)$ ,  $(6, 4)$ , and  $(7, 3)$

Look @ above graph

$$MRS(6, 3) = \text{undefined}$$

$$MRS(6, 4) = \text{undefined}$$

$$MRS(7, 3) = 0$$

- c) Find the  $MU_x(6, 3)$ ,  $MU_x(5, 3)$ ,  $MU_y(6, 3)$ ,  $MU_y(6, 2)$

$$\underline{MU_x(6, 3)}: u(6, 3) = \min\left\{\frac{1}{3}(6) + 1, 3\right\} = 3$$

$$u(7, 3) = \min\left\{\frac{1}{3}(7) + 1, 3\right\} = 3$$

$$\Rightarrow \text{as } x \uparrow \text{ by 1, utility } \uparrow \text{ by 0}$$

$$\Rightarrow MU_x(6, 3) = 0$$

$$\underline{MU_y(6, 3)}: \text{similar to } MU_x(6, 3)$$

$$\Rightarrow MU_y(6, 3) = 0$$

$$\underline{MU_x(5, 3)}: u(5, 3) = \min\left\{\frac{1}{3}(5) + 1, 3\right\} = \frac{8}{3}$$

$$u(6, 3) = \min\left\{\frac{1}{3}(6) + 1, 3\right\} = 3$$

$$\Rightarrow \text{as } x \uparrow \text{ by 1, utility } \uparrow \text{ by } 3 - \frac{8}{3} = \frac{1}{3}$$

$$\Rightarrow MU_x(5, 3) = \frac{1}{3}$$

$$\underline{MU_y(6, 2)}: u(6, 2) = 2$$

$$u(6, 3) = 3$$

$$\Rightarrow \text{as } y \uparrow \text{ by 1, utility } \uparrow \text{ by 1}$$

$$\Rightarrow MU_y(6, 2) = 1$$

Which of the following utility equations represent monotonic preferences?

- a)  $u(x, y) = xy$
- b)  $u(x, y) = x - y$
- c)  $u(x, y) = x^2 + y^{\frac{1}{8}}$
- d)  $u(x, y) = \frac{x}{y}$

a)  $\frac{\partial u}{\partial x} = y > 0$      $\frac{\partial u}{\partial y} = x > 0$     for all  $x > 0$  and  $y > 0$   
 $\Rightarrow$  Satisfies monotonicity

b)  $\frac{\partial u}{\partial y} = -1 < 0$     for all  $x > 0$  and  $y > 0$   
 $\Rightarrow$  does not satisfy monotonicity

c)  $\frac{\partial u}{\partial x} = 2x > 0$      $\frac{\partial u}{\partial y} = \frac{1}{8} y^{-7/8} > 0$     for all  $x > 0$  and  $y > 0$   
 $\Rightarrow$  Satisfies monotonicity

d)  $\frac{\partial u}{\partial y} = -\frac{x}{y^2} < 0$     for all  $x > 0$  and  $y > 0$   
 $\Rightarrow$  Satisfies monotonicity