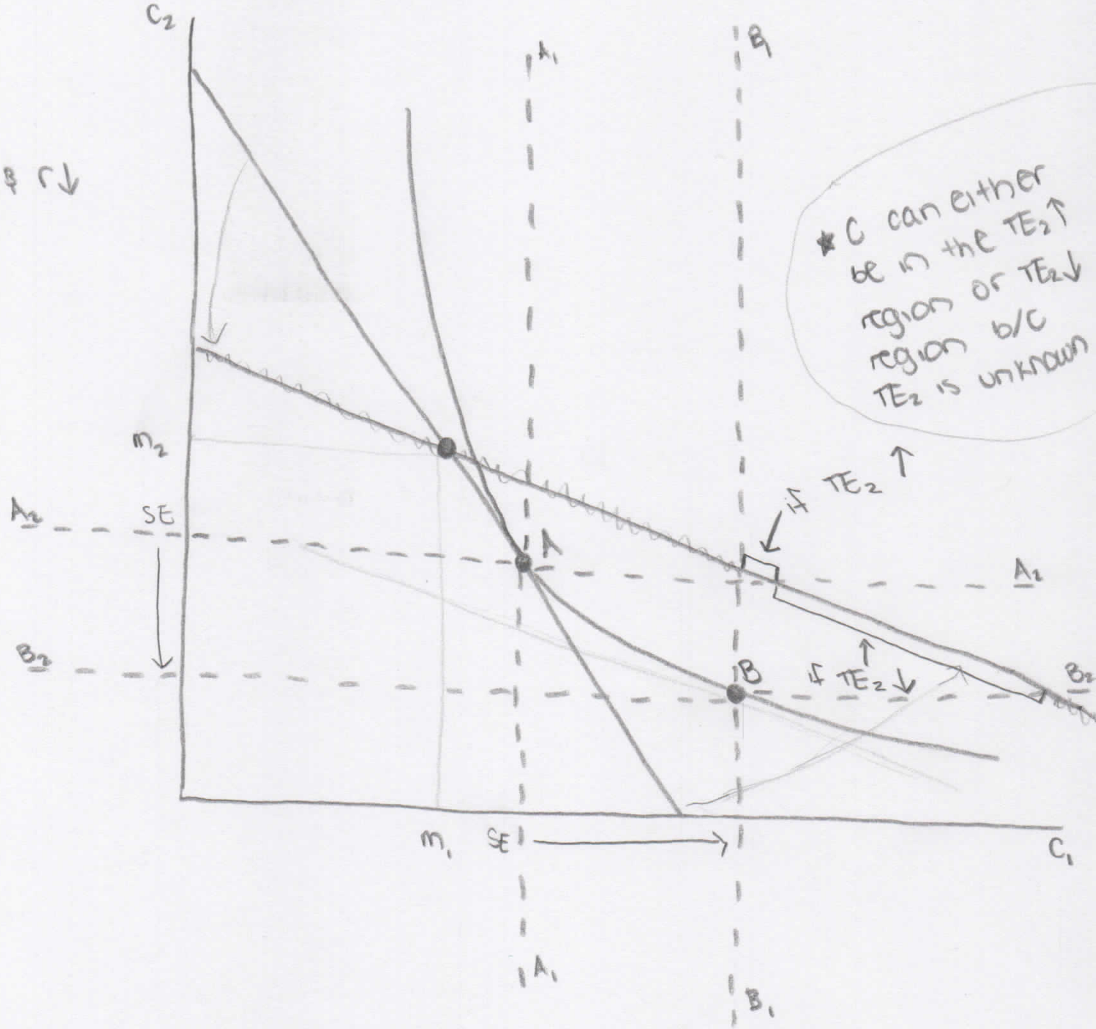
CLAS MIDTERM 2 REVIEW

1. Graph the substitution, income and total effects on c_1 and c_2 due to a decrease in the interest rate. What happens to savings? Assume they are a borrower.

	C_1	C_2
SE	↑	↓
IE	↑	↑
TE	↑	?

Borrower & $r\downarrow$ → P.P.↑



"crutch rule" for SE

$P_{C_1} \approx r$

$P_{C_2} \approx \pi$

savings

$S = m_1 - C_1$

$\Delta S = (\Delta m_1) - (\Delta C_1)$

$\Delta S = (0) - \underbrace{(\uparrow)}_{\text{from TE on } C_1}$ → $\boxed{\text{savings decrease}}$

2. $U(c_1, c_2) = 4c_1^{1/2} + c_2$

The interest rate is 100%, the deflation rate is 50%, and income in both period 1 and period 2 is \$2.

a. What is the real interest rate?

$\frac{1+r}{1+\pi} = 1+\rho$

$r = 100\% \rightarrow 1$

$\pi = -50\% \rightarrow -\frac{1}{2}$

$\frac{1+1}{1+(-\frac{1}{2})} = 1+\rho$

$\frac{2}{(\frac{1}{2})} = 1+\rho$

$4 = 1+\rho$

$\rho = 3$

$\boxed{\rho = 300\%}$

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b. Solve for c_1^* and c_2^*

① $U(c_1, c_2) = 4c_1^{1/2} + c_2$

② $MRS = \dfrac{2}{c_1^{1/2}}$

	$c_1 \uparrow$	$c_2 \downarrow$
MRS $\downarrow$	✓	—

DMRS → convex (use tangency)

③ $\dfrac{2}{c_1^{1/2}} = (1+\rho)$

④ $2 = c_1^{1/2}(1+\rho)$

$c_1^{1/2} = \dfrac{2}{1+\rho}$

$$\boxed{c_1^* = \frac{4}{(1+\rho)^2}}$$

→ $\dfrac{4}{4^2} = \dfrac{1}{4}$

⑤ $(1+\rho)c_1 + c_2 = (1+\rho)m_1 + m_2$

$(1+\rho)\left(\dfrac{4}{(1+\rho)^2}\right) + c_2 = (1+\rho)m_1 + m_2$

$\dfrac{4}{1+\rho} + c_2 = (1+\rho)m_1 + m_2$

$$\boxed{c_2^* = (1+\rho)m_1 + m_2 - \frac{4}{1+\rho}}$$

→ $4(2) + 2 - \dfrac{4}{4}$

$= 8 + 2 - 1$

$= 9$

c. Is this consumer a saver or a borrower? How much do they save or borrow?

$S = m_1 - c_1$

$= 2 - \frac{1}{4}$

$\boxed{S = \$1.75}$

savings are positive → $\boxed{\text{saver}}$

d. Find the numerical SE, IE, and TE on c_1 and c_2 if the inflation rate changes to 0%

	B.C. variables			Demands		
	$(1+\rho)$	m_1	m_2	c_1^*	c_2^*	U
A	4	2	2	$\frac{1}{4}$	9	11
B	2	X	X	1	7	11
C	2	2	2	1	4	

$U(\frac{1}{4}, 9) = 4(\frac{1}{4})^{1/2} + 9 = 4(\frac{1}{2}) + 9 = 11$

Point C

$c_1^* = \dfrac{4}{2^2} = 1$

$c_2^* = 2(2) + 2 - \dfrac{4}{2}$

$= 4$

Point B

$c_1^* = \dfrac{4}{2^2} = 1$

$U(c_1, c_2) = 4c_1^{1/2} + c_2$

$11 = 4(1)^{1/2} + c_2$

$11 = 4 + c_2$

$c_2^* = 7$

	c_1	c_2
SE = B − A	$1 - \frac{1}{4} = \frac{3}{4}$	$7 - 9 = -2$
IE = C − B	$1 - 1 = 0$	$4 - 7 = -3$
TE = C − A	$1 - \frac{1}{4} = \frac{3}{4}$	$4 - 9 = -5$

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3. Lucy is deciding how many hours to work. Her utility function is given by $U(R, C) = R^{3/4} C^{1/4}$, where R is the number of recreation hours per week and C is the dollars of consumption per week. The price of consumption is \$1 and Lucy can earn \$10 per hour at her job. In addition, Lucy's parents send her an allowance of \$400 per week. Assume that Lucy has 100 hours per week to allocate between working and recreation.

   a. Solve for R*, C*, and L*

① $U(R,C) = R^{3/4} C^{1/4}$

② $MRS = \frac{3C}{R}$

③ $\frac{3C}{R} = \frac{w}{p}$

④ $3Cp = wR$

$C = \frac{wR}{3p}$

⑥ $C = \frac{w}{3p}(R)$

$C = \frac{w}{3p}\left(\frac{3(M + w\bar{L})}{4w}\right)$

$\boxed{C^* = \frac{M + w\bar{L}}{4p}}$

$wR + pC = M + w\bar{L}$

⑤ $wR + p\left(\frac{wR}{3p}\right) = M + w\bar{L}$

$wR + \frac{wR}{3} = M + w\bar{L}$

$3wR + wR = 3(M + w\bar{L})$

$4wR = 3(M + w\bar{L})$

$\boxed{R^* = \frac{3(M + w\bar{L})}{4w}} \rightarrow 105$

R* cannot be greater than $\bar{L}$; must round down. If we are changing R, we must change C as well

$wR + PC = M + w\bar{L}$

$w\bar{L} + PC = M + w\bar{L}$

$PC = M$

$C^* = \frac{M}{P}$

$\boxed{R^* = \bar{L} = 100 \qquad C^* = \frac{M}{P} = 400 \qquad L^* = \bar{L} - R^* = 0}$

   b. Find the reservation wage

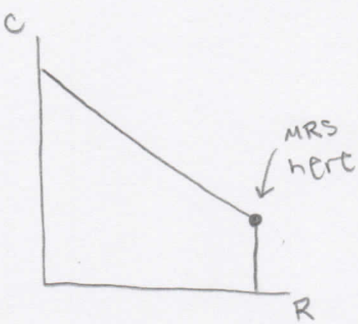
Res wage is the MRS when they don't work $(\bar{L}, \frac{M}{P})$

① $MRS = \frac{3C}{R}$

② $MRS(\bar{L}, \frac{M}{P}) = \frac{3\left(\frac{M}{P}\right)}{\bar{L}}$

$= \frac{3\left(\frac{400}{1}\right)}{100}$

$\boxed{\text{Res wage} = 12 \rightarrow \frac{12 \text{ units of C}}{1 \text{ hr of R}}}$



Notice that because the reservation wage is higher than the actual wage divided by price $\left(\frac{10}{1}\right)$, they choose not to work.

4. The face value of a bond is \$16, the coupon rate is 50%, the market interest rate is 100%, and the bond has two years until maturity. Determine the price (present value) of the bond.

$$PV = \underbrace{PV}_{\substack{\text{use market} \\ \text{int. rate}}} \text{ of } \underbrace{2 \text{ interest payments}}_{\text{Face value} \times \text{coupon rate}} + \underbrace{PV}_{\text{use market int. rate}} \text{ of face value}$$

$$PV = \frac{(\$16 \times 50\%)}{(1+1)^1} + \frac{(\$16 \times 50\%)}{(1+1)^2} + \frac{\$16}{(1+1)^2} = \frac{8}{2} + \frac{8}{4} + \frac{16}{4} = 4 + 2 + 4 = \boxed{\$10}$$

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5. Currently, my only wealth is from my house, which is worth $32. However, there is a 25% chance that my house will get damaged and lose half its value. I am considering buying insurance for my house, and I know that my insurance company will charge me $0.25 for every dollar of coverage I buy. I am allowed to buy coverage up to the amount of my loss (i.e. if my loss is $5, I can buy a maximum of $5 of coverage). If E[u($c_b$, c_g)] = ¼ ln(c_b) + ¾ ln(c_g) how much insurance coverage will I buy? In addition, what are my levels of consumption in the good state and in the bad state?

$\boxed{m = 32 \quad L = 16 \quad \gamma = \frac{1}{4}}$

① $E[c_b, c_g] = \frac{1}{4}\ln(c_b) + \frac{3}{4}\ln(c_g)$

② $MRS = \dfrac{c_g}{3c_b}$

③ $\dfrac{c_g}{3c_b} = \dfrac{\gamma}{1-\gamma}$

④ $c_g = \left(\dfrac{\gamma}{1-\gamma}\right)3c_b$

$\left(\dfrac{\gamma}{1-\gamma}\right)c_b + c_g = m + \left(\dfrac{\gamma}{1-\gamma}\right)(m-L)$

⑤ $\left(\dfrac{\gamma}{1-\gamma}\right)c_b + \left(\dfrac{\gamma}{1-\gamma}\right)3c_b = m + \left(\dfrac{\gamma}{1-\gamma}\right)(m-L)$

$4c_b\left(\dfrac{\gamma}{1-\gamma}\right) = m + \left(\dfrac{\gamma}{1-\gamma}\right)(m-L)$

$\boxed{c_b = \dfrac{m}{4}\left(\dfrac{1-\gamma}{\gamma}\right) + \dfrac{(m-L)}{4}}$

$\boxed{c_b^* = \dfrac{(1-\gamma)m + \gamma(m-L)}{4\gamma}} \rightarrow 28$

⑥ $c_g = \dfrac{3\cancel{\gamma}}{1-\gamma}\left(\dfrac{(1-\gamma)m + \gamma(m-L)}{4\cancel{\gamma}}\right)$

$\boxed{c_g^* = \dfrac{3[(1-\gamma)m + \gamma(m-L)]}{4(1-\gamma)}} \rightarrow 28$

$c_b^* = m - \gamma K$

$28 = 32 - \frac{1}{4}(K) \rightarrow \boxed{K^* = 16}$

6. Demand is: q = 16 – 4p

   a. Find elasticity of demand when p = $3 and when p = $0

$\varepsilon_{q,p} = \underbrace{\dfrac{\partial q}{\partial p}} \cdot \dfrac{p}{q}$

$\dfrac{\partial(16-4p)}{\partial p} = -4$

$\varepsilon_{q,p} = \dfrac{-4p}{q}$

$\boxed{\varepsilon_{q,p} = \dfrac{-4p}{16-4p}}$

p = $3

$\varepsilon = \dfrac{(-4 \times 3)}{16-(4\times 3)}$

$= \dfrac{-12}{16-12}$

$\boxed{\varepsilon = -3}$

p = $0

$\varepsilon = \dfrac{(-4\times 0)}{16-(4\times 0)}$

$\boxed{\varepsilon = 0}$

b. At what price does elasticity of demand equal -1?

$\varepsilon_{q,p} = \dfrac{-4p}{16-4p}$

$-1 = \dfrac{-4p}{16-4p}$

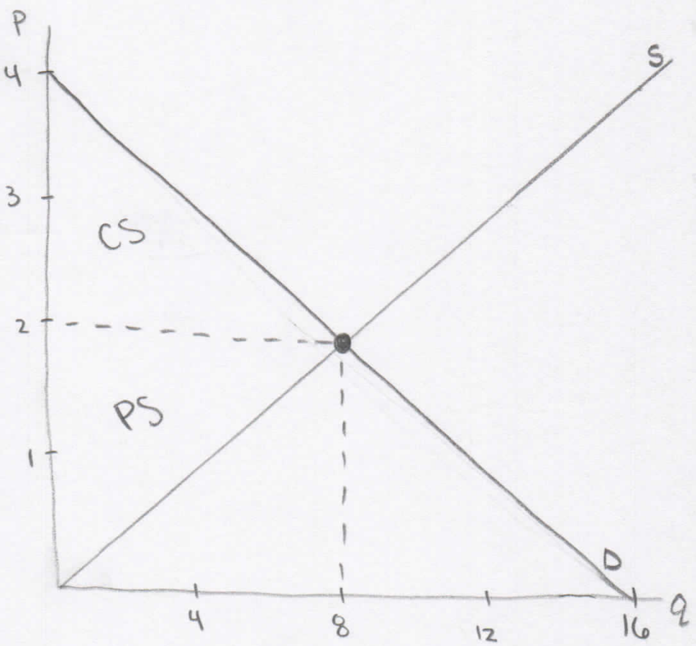
$-1(16-4p) = -4p$

$16-4p = 4p$

$16 = 8p$

$\boxed{p = \$2}$

c. Supply is: q = 4p. Find equilibrium price and quantity. In addition, find CS, PS, and total welfare.



Demand = Supply

$16 - 4p = 4p$

$16 = 8p$

$\boxed{p = \$2}$

$q = 16 - 4p$

$q = 16 - (4 \times 2)$

$\boxed{q = 8}$

$q = 4p \rightarrow q = (4 \times 2) \rightarrow q = 8$ ✓

$CS = \frac{1}{2}(2)(8)$

$\boxed{CS = \$8}$

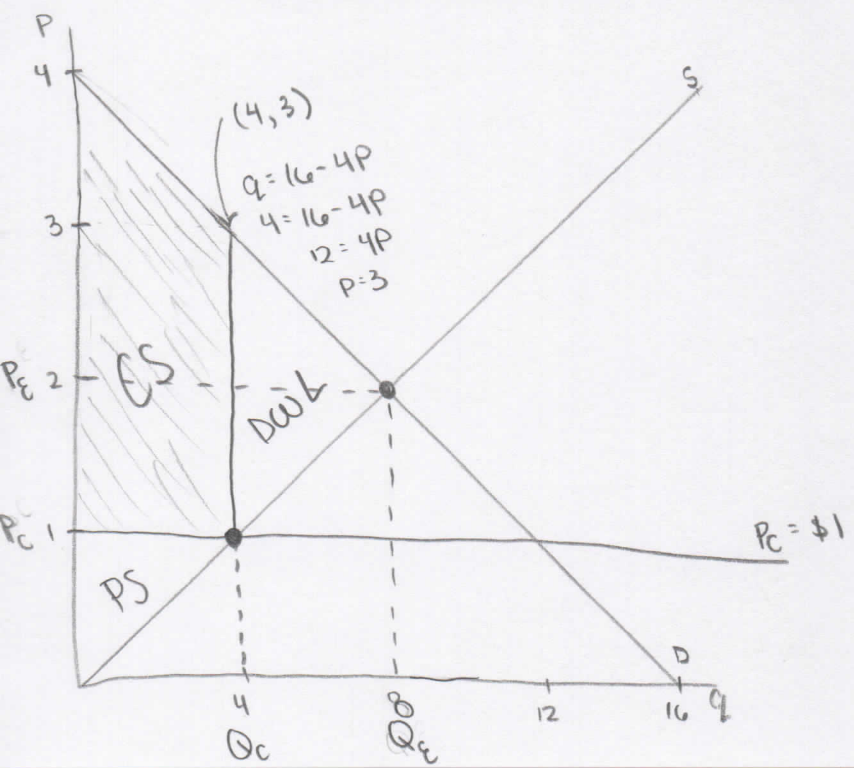
$PS = \frac{1}{2}(2)(8)$

$\boxed{PS = \$8}$

$TW = CS + PS$

$\boxed{TW = \$16}$

d. The government sets a price ceiling at \$1. What is the new quantity? In addition, find CS, PS, total welfare, and deadweight loss.



$q_c = 4P_c$

$q_c = 4(1)$

$\boxed{q_c = 4}$

$q_c = 16 - 4P_c$

$q_c = 16 - 4(1)$

$q_c = 12$ X

always choose lower of two quantities

$CS = (2 \times 4) + \frac{1}{2}(1)(4) = 8 + 2 = \boxed{\$10}$

$PS = \frac{1}{2}(1)(4) = \boxed{\$2}$

$TW = CS + PS = 10 + 2 = \boxed{\$12}$

$DWL = \frac{1}{2}(2)(4) = \boxed{\$4}$

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$\to q = 40 - p$ $\qquad$ $\to q = 30 - p$

7. Consumer A: $p = 40 - q$ Consumer B: $p = 30 - q$

   a. Find the equation for market demand

Ⓐ pays up to $40; Ⓑ pays up to $30. Both will buy if $0 \le p < 30$. Only Ⓐ will buy if $30 \le p < 40$. No one will buy if $p \ge 40$.

* If both buy

$q = 40 - p$

$+ \; q = 30 - p$

$q = 70 - 2p$

If $p \ge 40$ then $q = 0$ (no one buys)

If $30 \le p < 40$ then $q = 40 - p$ (only Ⓐ buys)

If $0 \le p < 30$ then $q = 70 - 2p$ (Ⓐ and Ⓑ buy)

$\to q = 10 - 2p$

   b. Consumer C: $p = 5 - (q/2)$

   Find the equation for market demand (using consumers A, B, and C)

Ⓐ pays up to $40; Ⓑ pays up to $30; Ⓒ pays up to $5. All three will buy if $0 \le p < 5$. Only Ⓐ and Ⓑ will buy if $5 \le p < 30$. Only Ⓐ will buy if $30 \le p < 40$. No one will buy if $p \ge 40$.

* If Ⓐ and Ⓑ buy

$q = 70 - 2p$

* If all three buy

$q = 40 - p$

$+ \; q = 30 - p$

$+ \; q = 10 - 2p$

$q = 80 - 4p$

If $p \ge 40$ then $q = 0$ (no one buys)

If $30 \le p < 40$ then $q = 40 - p$ (only Ⓐ buys)

If $5 \le p < 30$ then $q = 70 - 2p$ (Ⓐ and Ⓑ buy)

If $0 \le p < 5$ then $q = 80 - 4p$ (all three buy)