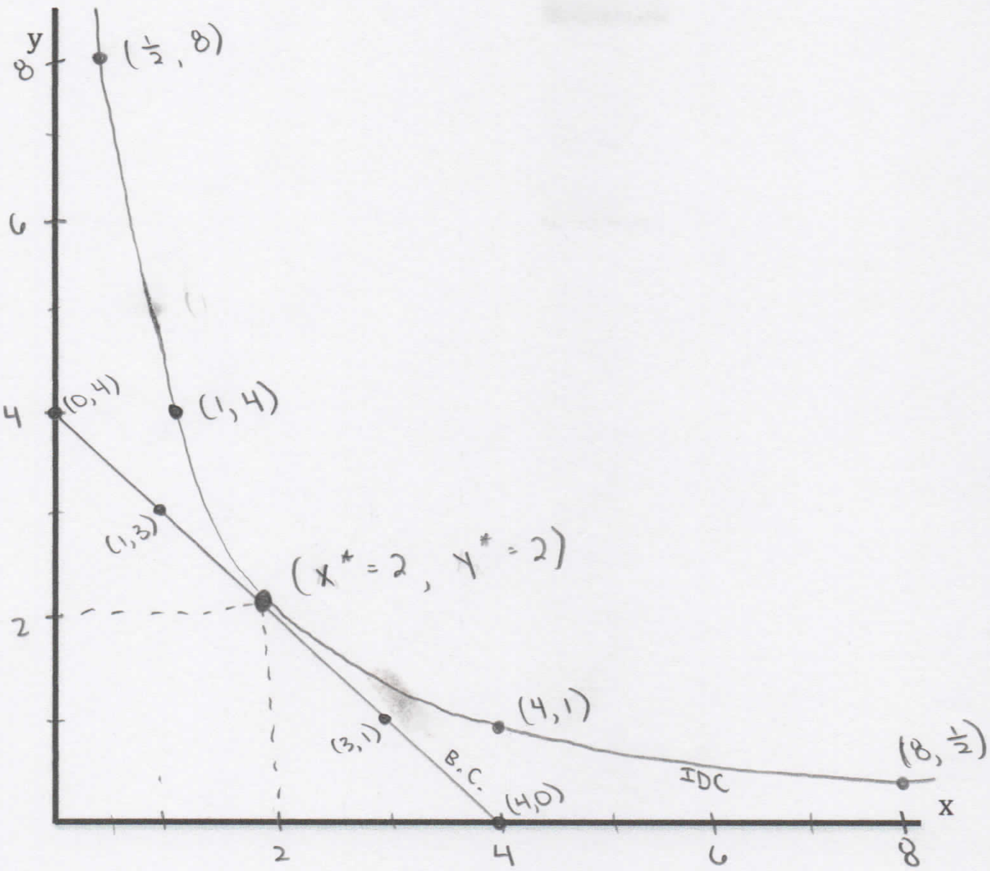
Consumer Optimization Fundamentals

The consumer optimization processes we use in this class find the point that brings the most utility by simultaneously testing all points along the budget constraint—following these processes is the fastest way to get to the answer. The goal of this worksheet, however, is to understand the concepts behind the processes. In each problem, you will work with a particular type of utility function, testing each point along the budget constraint individually to find the one that brings the most utility. You will then extract a general pattern from each answer, which will deepen your understanding of the step-by-step processes. Note: In general, consumers are allowed to consume fractions of goods, but to make things simple we will assume on this worksheet that they can only consume in whole numbers.

I) $u(x, y) = x * y$ $\quad m = \$4 \quad p_x = \$1 \quad p_y = \$1$

a) This is a Cobb-Douglas utility function

b) Graph the budget constraint and label each whole-number coordinate (i.e. label (10, 10) not (8.5, 8.5); there will be five points)



Budget Constraint

$P_x X + P_y Y = m$

$X + Y = 4$

X-intercept = 4

Y-intercept = 4

IDC

$u(x, y) = XY$

$4 = XY$

utility at optimum

$Y = \frac{4}{X}$

X	Y
$\frac{1}{2}$	8
1	4
2	2
4	1
8	$\frac{1}{2}$

c) Test utility at each of the five points (continued on the next page; the first line is already filled out for you)

POINT	X	*	Y	=	U
(0 , 4)	0	*	4	=	0
(1 , 3)	1	*	3	=	3
(2 , 2)	2	*	2	=	4

(3 , 1)	3 *	1	= 3
(4 , 0)	4 *	0	= 0

	$x\uparrow$	$y\downarrow$
MRS $\downarrow$	✓	✓

d) GENERAL PATTERNS

i) Utility is maximized at (2 , 2)

ii) The MRS as a function of x and y = y/x. The MRS is {diminishing, constant, or increasing}.

iii) At optimum (2 , 2), MRS = $\frac{2}{2} = 1$ and p_x / p_y = $\frac{1}{1} = 1$

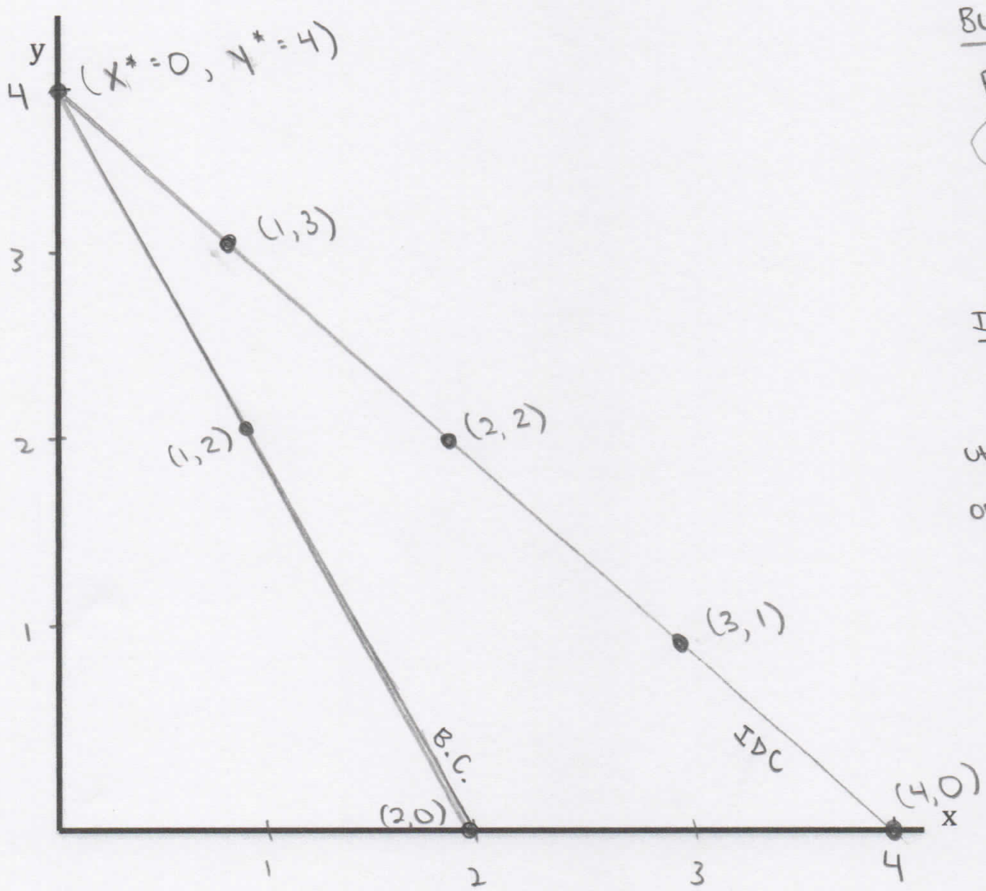
iv) Thus, when utility is maximized for a Cobb-Douglas type of utility function, MRS { >, <, or = } p_x / p_y * known as the "tangency condition"

e) On your graph from part b, draw the IDC on which the utility maximizing point lies (Hint: all points on the IDC will have the same utility)

II) $u(x, y) = x + y$ $m = \$4$ $p_x = \$2$ $p_y = \$1$

a) This is a Perfect Subs/Linear utility function

b) Graph the budget constraint and label each whole-number coordinate (there will be three points)



Budget constraint

$P_x x + P_y y = m$

$2x + y = 4$

x-intercept = 2

y-intercept = 4

IDC

$u(x, y) = x + y$

utility at optimum $\leftarrow$ $4 = x + y$

$y = 4 - x$

x	y
0	4
1	3
2	2
3	1
4	0

c) Test utility at each of the three points

POINT	X	+	Y	=	U
(0 , 4)	0	+	4	=	4
(1 , 2)	1	+	2	=	3
(2 , 0)	2	+	0	=	2

d) GENERAL PATTERNS

i) Utility is maximized at (0 , 4)

ii) The MRS as a function of x and y = $\frac{1}{1} = 1$. The MRS is {diminishing, constant, or increasing}.

iii) At optimum (0 , 4), MRS = 1 and $p_x / p_y = \frac{2}{1} = 2$

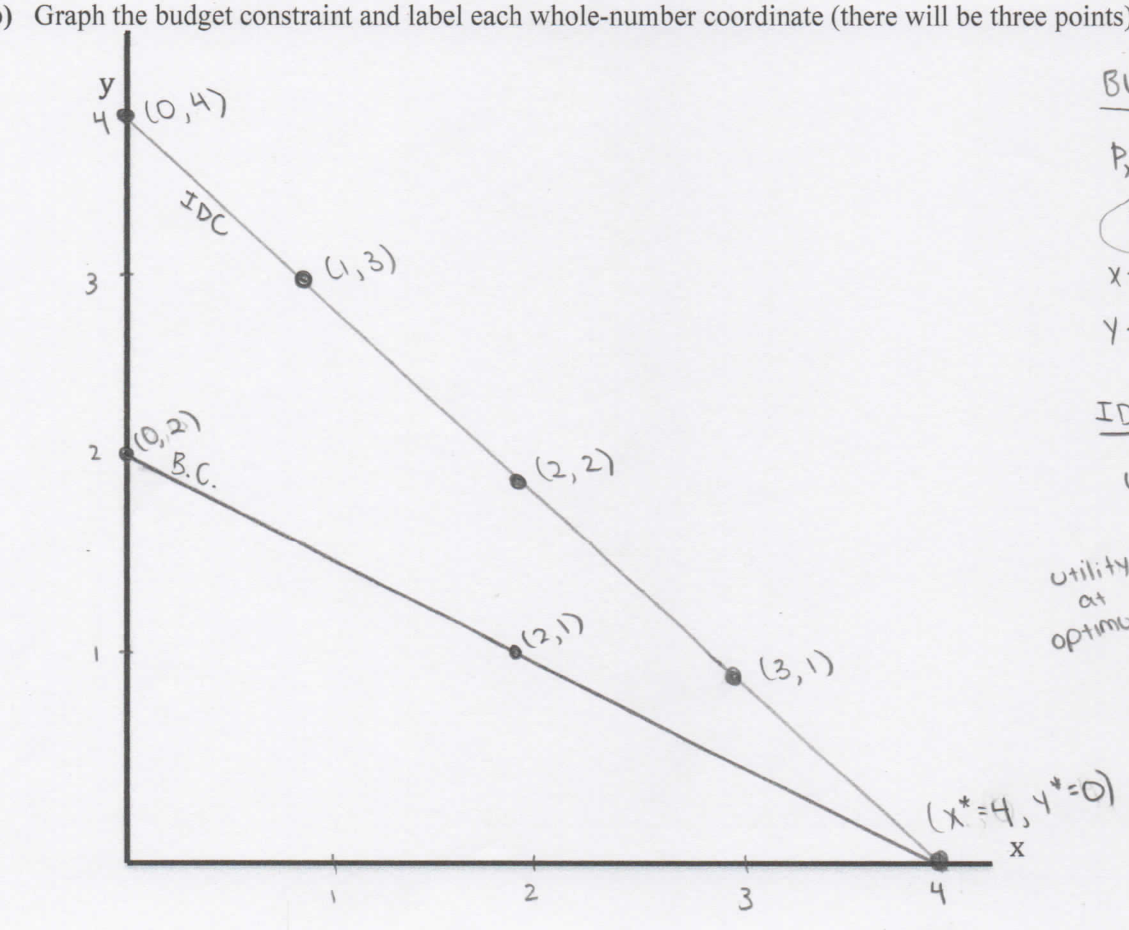
iv) Thus, for a perfect subs type of utility function, if MRS { >, <, or =} p_x / p_y, then the consumer chooses {all x, all y, or indifferent}

e) On your graph from part b, draw the IDC on which the utility maximizing point lies (Hint: all points on the IDC will have the same utility)

III) u(x, y) = x + y **m = \$4** **p_x = \$1** **p_y = \$2**

a) This is a Perfect subs / Linear utility function

b) Graph the budget constraint and label each whole-number coordinate (there will be three points)



Budget Constraint

$P_x x + P_y y = m$

$x + 2y = 4$

x-intercept = 4

y-intercept = 2

IDC

$u(x, y) = x + y$

$4 = x + y$

utility at optimum

$y = 4 - x$

x	y
0	4
1	3
2	2
3	1
4	0

c) Test utility at each of the three points

POINT	X	+	Y	=	U
(0 , 2)	0	+	2	=	2
(2 , 1)	2	+	1	=	3
(4 , 0)	4	+	0	=	4

d) GENERAL PATTERNS

i) Utility is maximized at (4 , 0)

ii) The MRS as a function of x and y = $\frac{1}{1} = 1$. The MRS is {diminishing, constant, or increasing}.

iii) At optimum (4 , 0), MRS = 1 and p_x / p_y = $\frac{1}{2}$

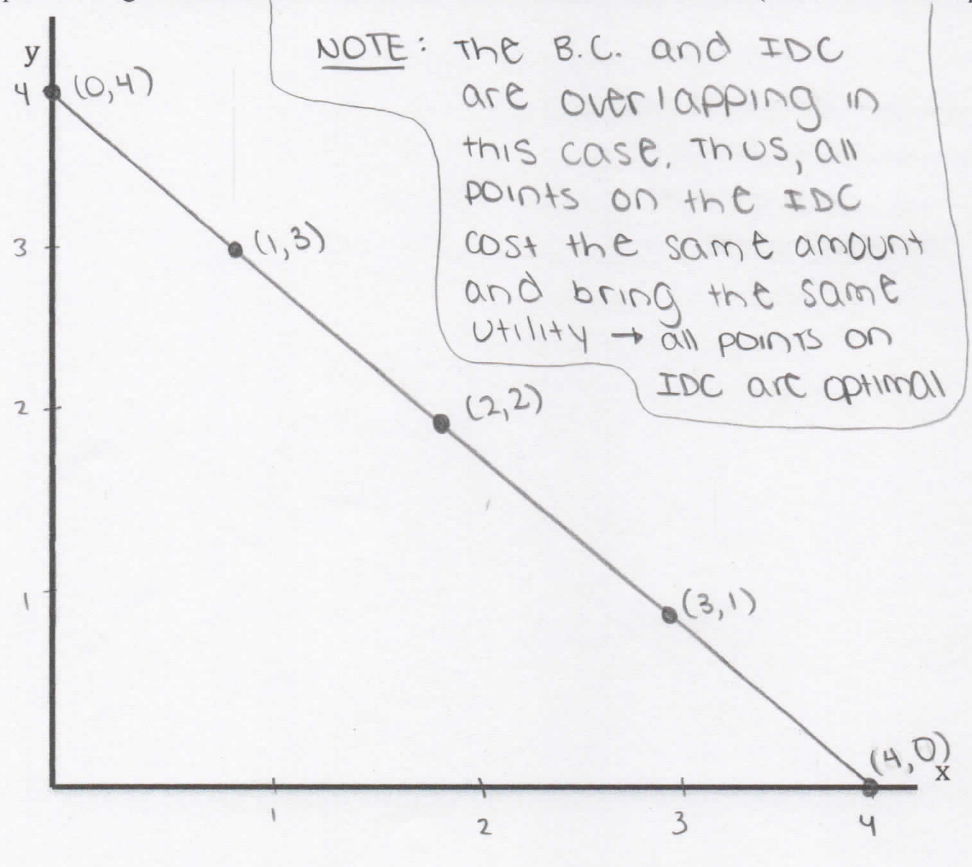
iv) Thus, for a Perfect Subs type of utility function, if MRS {>, <, or =} p_x / p_y, then the consumer chooses {all x, all y, or indifferent}

e) On your graph from part b, draw the IDC on which the utility maximizing point lies (Hint: all points on the IDC will have the same utility)

IV) u(x, y) = x + y **m = \$4** **p_x = \$1** **p_y = \$1**

a) This is a Perfect Subs/Linear utility function

b) Graph the budget constraint and label each whole-number coordinate (there will be five points)



NOTE: The B.C. and IDC are overlapping in this case. Thus, all points on the IDC cost the same amount and bring the same utility → all points on IDC are optimal

Budget Constraint

$P_x X + P_y Y = m$

$X + Y = 4$

x-intercept = 4

y-intercept = 4

IDC

$U(x,y) = x + y$

utility at optimum ← $4 = x + y$

x	y
0	4
1	3
2	2
3	1
4	0

c) Test utility at each of the three points

POINT	X	+	Y	=	U
(0 , 4)	0	+	4	=	4
(1 , 3)	1	+	3	=	4
(2 , 2)	2	+	2	=	4
(3 , 1)	3	+	1	=	4
(4 , 0)	4	+	0	=	4

d) GENERAL PATTERNS

i) Utility is maximized at all affordable points

ii) The MRS as a function of x and y = $\frac{1}{1} = 1$. The MRS is {diminishing, constant, or increasing}.

iii) At all affordable points MRS = 1 and p_x / p_y = 1

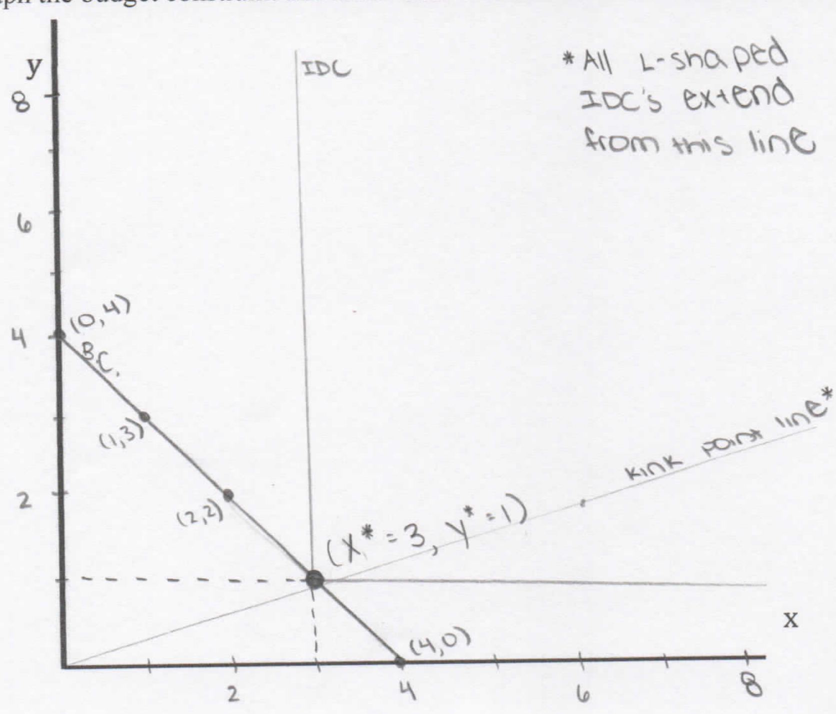
iv) Thus, for a perfect subs type of utility function, if MRS { >, <, or = } p_x / p_y, then the consumer chooses {all x, all y, or indifferent}

e) On your graph from part b, draw the IDC on which the utility maximizing point lies (Hint: all points on the IDC will have the same utility)

V) $u(x, y) = \min [x , 3y]$ **m = $4** **$p_x$ = $1** **$p_y$ = $1**

a) This is a Perfect Comps/Leontief utility function

b) Graph the budget constraint and label each whole-number coordinate (there will be five points)



Budget Constraint

$P_x x + P_y y = m$

$x + y = 4$

x-intercept = 4

y-intercept = 4

IDC

$u(x,y) = \min\underbrace{[x, 3y]}$

$x = 3y$

$y = \frac{x}{3}$

This is the equation of all the kink points

c) Test utility at each of the five points

POINT	min[X ,	Y]	=	U
(0 , 4)	min[0 ,	(3 * 4)]	=	0
(1 , 3)	min[1 ,	(3 x 3)]	=	1
(2 , 2)	min [2 ,	(3 x 2)]	=	2
(3 , 1)	min [3 ,	(3 x 1)]	=	3
(4 , 0)	min [4 ,	(3 x 0)]	=	0

d) GENERAL PATTERNS

i) Utility is maximized at (3 , 1)

ii) For a perfect comps type of utility function, goods are always consumed in a fixed ratio. In this case, utility is maximized when 1 * x = 3 * y. (Now, compare the ratio statement to the original utility function and see if you notice any patterns.) The general formula for a Perfect comps type of utility function is u(x, y) = A * min [(x / a) , (y / b)]. Thus, in general, utility for a perfect comps type of function is maximized when $\frac{1}{a}$ * x = $\frac{1}{b}$ * y.

e) On your graph from part b, draw the IDC on which the utility maximizing point lies (Hint: all points on the IDC will have the same utility)