

Cobb-Douglas

Linear/Perfect Substitutes

Leontief/Perfect Complements

Quasilinear—Convex

Quasilinear—Concave

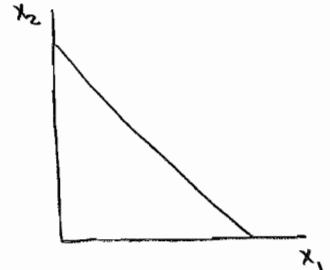
I. Identifying types of utility

Match each of the utility functions below with one of the types of utility listed in the word bank above (some types may be used more than once). Then, in the space provided below each function, describe how you determined which type of utility the function has.

1. $U(x_1, x_2) = x_1 + (1/2)x_2$

TYPE: Linear / Perfect Substitutes

- Two goods being added together
- Both goods have an exponent of 1
- No natural logs
- Constant MRS
- Monotonic

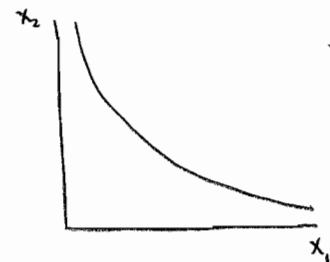


2. $U(x_1, x_2) = x_1 + \ln(x_2)$

TYPE: Quasilinear - Convex

- Two goods being added together
- Natural log
- Diminishing MRS \rightarrow convex

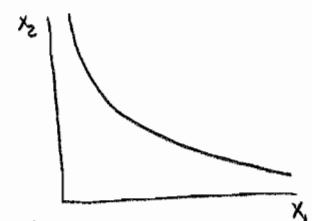
$$MRS = \frac{1}{x_2} = x_2 \quad x_1 \uparrow x_2 \downarrow \Rightarrow MRS \downarrow$$



3. $U(x_1, x_2) = x_1^{1/2} x_2^3$

TYPE: Cobb-Douglas

- Two goods being multiplied together with positive exponents
- Monotonic
- Convex



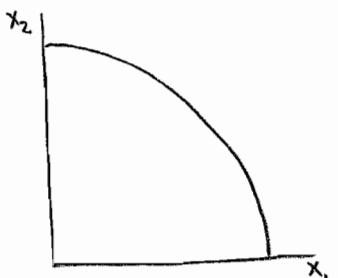
4. $U(x_1, x_2) = x_1^2 + x_2$

TYPE: Quasilinear - Concave

- Two goods being added together
- One good has an exponent not equal to 1
- Increasing MRS \rightarrow concave

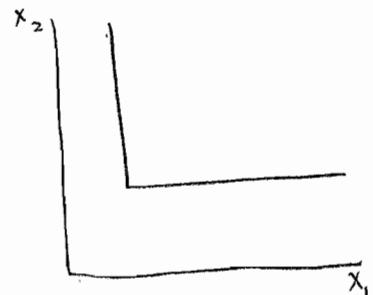
$$MRS = 2x_1$$

$$x_1 \uparrow x_2 \downarrow \Rightarrow MRS \uparrow$$



5. $U(x_1, x_2) = \min [2x_1, 3x_2]$ TYPE: Perfect Complements / Leontief

- The function has a "min" in it



6. $U(x_1, x_2) = x_1^{1/2} + x_2$ TYPE: Quasilinear - Convex

- Two goods being added together
- One good has an exponent not equal to 1
- Diminishing MRS \rightarrow convex

$$MRS = \frac{1}{2} x_1^{-\frac{1}{2}} = \frac{1}{2x_1^{\frac{1}{2}}} \quad x_1 \uparrow x_2 \downarrow \Rightarrow MRS \downarrow$$



7. $U(x_1, x_2) = x_1 x_2$ TYPE: Cobb-Douglas

See #3

II. Marginal Rate of Substitution

For the following questions, use the function $U(x_1, x_2) = 4x_1^{1/2} x_2^{1/2}$

1. Find the MRS

$$MRS = \frac{MU_1}{MU_2} = \frac{4(\frac{1}{2})x_1^{-\frac{1}{2}}x_2^{\frac{1}{2}}}{4(\frac{1}{2})x_1^{\frac{1}{2}}x_2^{-\frac{1}{2}}} = \frac{2x_2^{\frac{1}{2}-(-\frac{1}{2})}}{2x_1^{\frac{1}{2}-(-\frac{1}{2})}} = \boxed{\frac{x_2}{x_1}}$$

2. Find the MRS at the point (1, 1)

$$MRS(1, 1) = \frac{1}{1} = \boxed{1}$$

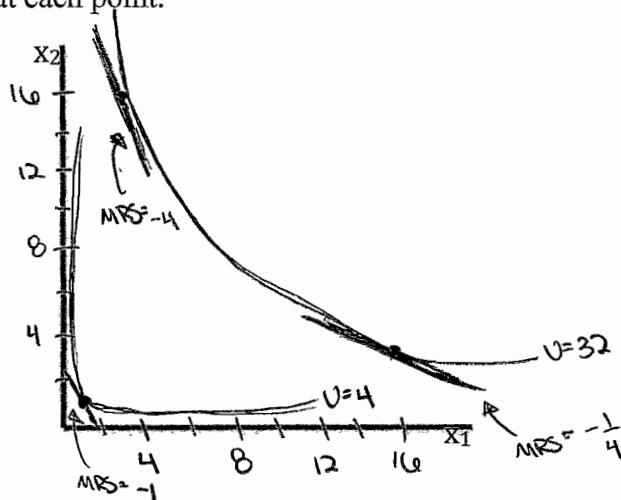
3. Find the MRS at the point (4, 16)

$$MRS(4, 16) = \frac{16}{4} = \boxed{4}$$

4. Find the MRS at the point (16, 4)

$$MRS(16, 4) = \frac{4}{16} = \boxed{\frac{1}{4}}$$

5. Sketch a graph showing the three points listed above, the indifference curve(s) that they're on, and the MRS at each point.



$U(4, 16) = U(16, 4) \Rightarrow$ same
indifference
curve

$U(1, 1) < U(4, 16) \Rightarrow$ lower
indifference
curve

Leontief-Douglas \Rightarrow convex

$$\begin{aligned} &\underline{(1, 1)} \\ &U(1, 1) = 4(1^{\frac{1}{2}})(1^{\frac{1}{2}}) \\ &\quad = 4 \\ &MRS(1, 1) = 1 \end{aligned}$$

$$\begin{aligned} &\underline{(4, 16)} \\ &U(4, 16) = 4(4^{\frac{1}{2}})(16^{\frac{1}{2}}) \\ &\quad = 4(2)(4) \\ &\quad = 32 \\ &MRS(4, 16) = 4 \end{aligned}$$

$$\begin{aligned} &\underline{(16, 4)} \\ &U(16, 4) = 4(16^{\frac{1}{2}})(4^{\frac{1}{2}}) \\ &\quad = 4(4)(2) \\ &\quad = 32 \\ &MRS(16, 4) = \frac{1}{4} \end{aligned}$$