Problem Set. Meeting 8.

Problem 1

$\mathcal{U}(A,B) = \min\{8A, 2B\}$

$p_A = 10$, $p_B = 5$, $m = 100$

Find the optimal consumption bundle.

Problem 2

$\mathcal{U}(a,b) = \frac{1}{3}a + 2b$

$p_a = 5$. $p_b = 30$. $m = 100$.

Find the optimal consumption bundle.

Problem 3

$\mathcal{U}(x_1, x_2) = 4 \cdot \sqrt{x_1} + x_2$

$p_1 = 1$. $p_2 = 2$. $M = 9$.

Find the optimal consumption bundle.

Problem 4

$\mathcal{U}(x,y) = \max\{\frac{1}{8}x, \frac{1}{16}y\}$

$p_x = 2$. $p_y = 1$. $m = 10$.

Find the optimal consumption bundle.

Problem 5

$\mathcal{U}(x,y) = x + y^4$

$p_x = 2$. $p_y = 10$. $m = 100$

Find the optimal consumption bundle.

Problem 6

$\mathcal{U}(c,w) = c + 5w$

$p_c = 5$. $p_w = 10$. $m = 100$.

4 coupons giving 25% off a bottle of whiskey.

Find the optimal consumption bundle.

Problem Set. Meeting 3. Solutions.

Problem 1

We set $8A = 2B$ which gives $B = 4A$

In the BC, this gives: $p_A \cdot A + p_B \cdot B = m$

$$p_A \cdot A + p_B \cdot 4A = m$$

$$A(p_A + 4p_B) = m$$

$$A = \frac{m}{p_A + 4p_B} = \frac{100}{10+20} = \frac{100}{30} = \frac{10}{3}$$

and so $B = \frac{4 \cdot m}{p_A + 4p_B} = \frac{400}{30} = \frac{40}{3}$

Problem 2

$MRS = -\frac{1/3}{2} = -\frac{1}{6}$ and $-\frac{p_1}{p_2} = \frac{-5}{30} = -\frac{1}{6}$. so all bundles are optimal bundles.

Problem 3

$u = 4 \cdot x_1^{1/2} + x_2$ so $MRS = -\frac{2 \cdot x_1^{-1/2}}{1} = -\frac{2}{x_1^{1/2}}$. dim MRS? as $x_1\uparrow$, $|MRS|\downarrow$. so yes. so interior solution. so $|MRS| = |-\frac{p_1}{p_2}|$

so $\frac{2}{x_1^{1/2}} = \frac{p_1}{p_2}$ so $x_1^{1/2} = \frac{p_2}{p_1} \cdot 2$ so $x_1 = \left(\frac{2p_2}{p_1}\right)^2$

so $x_1 = 2\left(\frac{2}{1}\right)^2 = 16$.

demand for x_2? $p_1x_1 + p_2x_2 = m$ so $p_1 \cdot 4 \cdot \frac{p_2^2}{p_1^2} + p_2 \cdot x_2 = m$

so $\frac{4p_2^2}{p_1} + p_2 \cdot x_2 = m$

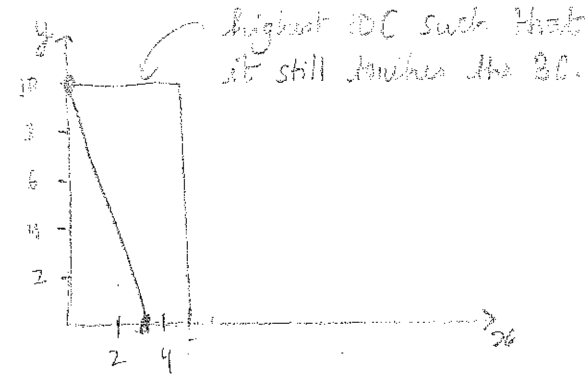
so $p_2 x_2 = m - \frac{4p_2^2}{p_1}$

so $x_2 = \frac{m}{p_2} - \frac{4p_2}{p_1}$

so $x_2 = -35$. which isn't possible at the current BC. we take the closest possibility to this, which is $x_2 = 0$, and $x_1 = 9$.

Problem 4

$\mathcal{U} = \max\left[\frac{1}{8}x, \frac{1}{16}y\right] \Rightarrow \max\{2x, y\}$. this gives



try drawing a budget constraint such that it reaches the highest IDC.

BC: $3x + y = 10$

so graphically, the highest IDC which touches the IDC is at $y=10$.
so optimal bundle: $(0, 10)$

Problem 5

$\mathcal{U} = x + y^4$

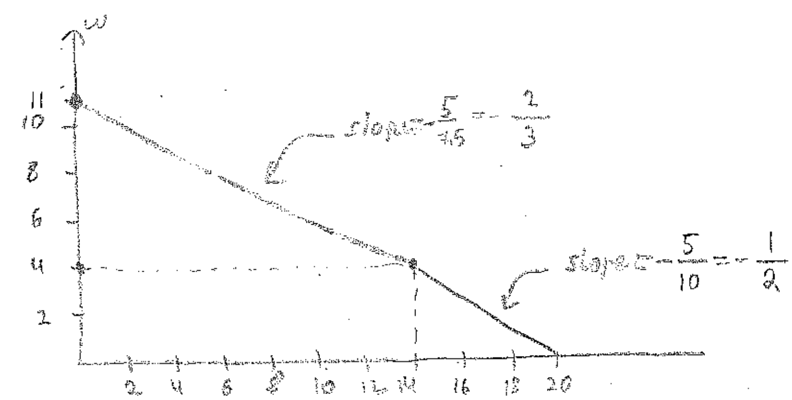
BC: $2x + 10y = 100$

we have $MRS = -\frac{1}{4y^3}$. as $y \downarrow$, $|MRS| \uparrow$ so not dim. MRS. so not interior solution.
So boundary solution.

so we compare utilities at $(50, 0)$ and $(0, 10)$.
$\mathcal{U}(50, 0) = 50$. $\mathcal{U}(0, 10) = 10^4$. $\mathcal{U}(0,10) > \mathcal{U}(50,0)$, so the optimal bundle is $(0,10)$

Problem 6

let's draw the BC.



if $w=4$, we spend $7.5 \cdot 4 = 30$
we still have \$70.
if we spend it all on whiskey, we get an additional 7 whiskeys. $\max(w) = 11$
if we spend it all on cigarettes, we get $\frac{70}{5} = 14$ cigarettes.

if we spend all our income on cigarettes ($w=0$), then we consume $\frac{100}{5} = 20$ cigarettes.

and $MRS = -\frac{1}{5}$, flatter than the slopes of the budget constraint.
so optimal bundle is $(0, 11)$.

Check: $\mathcal{U}(0,11) = 55$. $\mathcal{U}(14,4) = 14 + 20 = 34$. $\mathcal{U}(20,0) = 20$.