Profit Max and RTS

This handout includes 3 sections: calculating returns to scale, the Impact of RTS on profit max, and an application problem. In this handout, I assume constant price, wage, and rent. I STRONGLY recommend checking out the "Profit Max Practice Problems" handout after you finish this one. Solutions to 1) and 3) are at the end of the worksheet.

1) Calculating Returns to Scale

Compare F(**t**L, **t**K) to **t**F(L,K) (where t>1)

Which is the same as comparing doubling[1] inputs to doubling outputs

What are the returns to scale for the following?

A) $F(L,K) = L^{1/2}K^{1/2}$

If I double inputs, outputs increase by ______________ double. This means I have ____________ returns to scale (I doubled my labor force and my capital, and the # of hotdogs produced increased by _________ double)

B) $F(L,K) = LK$

If I double inputs, outputs increase by ______________ double. This means I have ____________ returns to scale (I doubled my labor force and my capital, and the # of hotdogs produced increased by _________ double)

C) $F(L,K) = \text{Min}(2L, K)$

If I double inputs, outputs increase by ______________ double. This means I have ____________ returns to scale (I doubled my labor force and my capital, and the # of hotdogs produced increased by _________ double)

D) $F(L,K) = \text{Min}(L^{1/2}, K)$

If I double inputs, outputs increase by ______________ double. This means I have ____________ returns to scale (I doubled my labor force and my capital, but the # of hotdogs produced increased by _________ double)

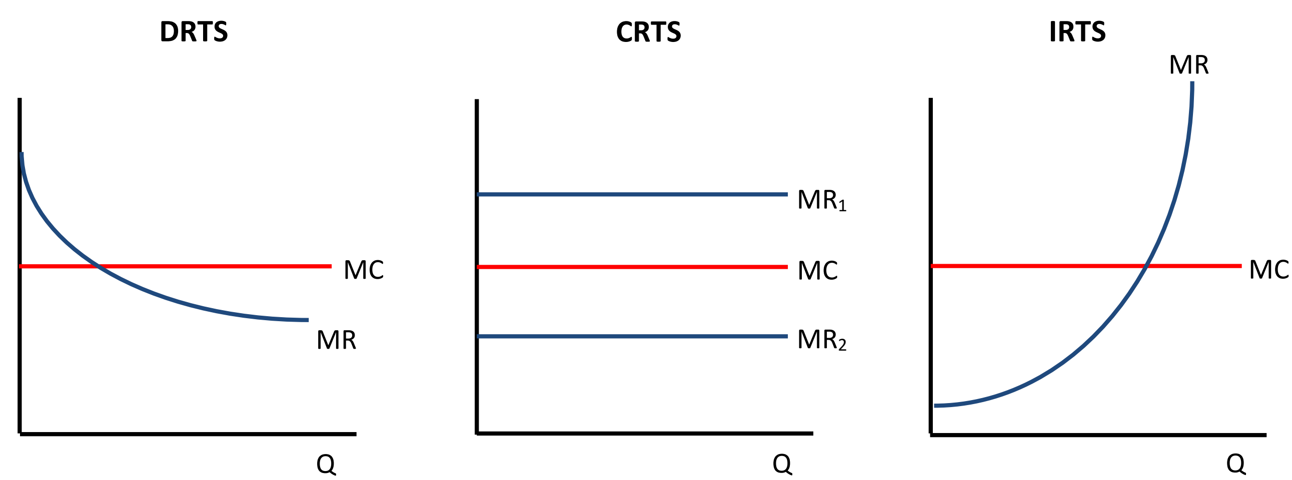
E) What is the exponent rule for returns to scale with Cobb-Douglas? (check the slides)

[1]If we increase inputs by "t," what happens to output? If t is 2, are doubling our inputs. However, it could easily be 3, 4, etc. I say double (2 for t) because it is the easiest to talk about and explain, but it could easily be tripling, quadrupling, etc.

2) Impact of RTS on profit max

In profit max problems, we are trying to maximize our profit (π) equation: P*F(L,K) -wL -rK. P*F(L,K) is our revenue (F(L,K)=Q, so P*F(L,K) is price times quantity) and wL + rK are our costs. If we take the derivative of this equation in terms of labor, we get P*MPL -w = 0 which is simplified to P*MPL=w. P*MPL is the derivative of revenue in terms of labor, giving us the marginal revenue (MR) from one more unit of labor. This makes sense since MPL is the amount of units the next worker will be able to make, and price is how much we can sell those units for. w is derivative of costs in terms of labor, which gives us the marginal cost (MC) of one more unit of labor. This makes sense because the cost of 1 more worker is the wage we must pay them. The same process applies to capital, giving us the MR of capital (P*MPK) and the MC of capital (r).

The condition we normally use to solve for profit max is P*MPL=w and P*MPK=k. Basically, we take the derivative of the profit equation, which gives us MR and MC, then set MR equal to MC. **However, this only works with DRTS!** Why? RTS tells us about MPL or MPK. If we are looking at labor, IRTS means MPL is increasing, CRTS means MPL is constant, and DRTS means MPL is decreasing. Since P*MPL is MR, **MR is increasing for IRTS, constant for CRTS, and decreasing for DRTS** (since we assume price is always constant). MC is constant, since wage and rent are constant (unless there is some sort of twist added to the question). These facts are best illustrated by the following graphs:



DRTS: **Consume where P*MPL=w or P*MPK=r (MR=MC).** At MR=MC, we have taken advantage of all of the points where MR>MC. Beyond this point, MR < MC. If we hire/rent beyond this point, we will start to lose money.

CRTS: **Compare P*MPL VS w and P*MPK VS r. Hire as much as possible if P*MPL > w, and hire no one if P*MPL < w (the same goes for capital).** MR is constant, so MR is either always above MC (P*MPL>w or P*MPK>r) or always below MC (P*MPL<w or P*MPK<r). If P*MPL > w, we will make a constant profit on every worker, so hire as many as possible. If P*MPL < w, we will make a loss on every worker, so hire no one. Expect there to be limits in these types of problems (Ex you can only hire 5 workers, or after 5 workers the wage goes up) since the professors tend to avoid infinite answers. It is also possible that MR=MC. If this happens, you would be indifferent between hiring and not hiring workers/renting machines.

IRTS: **Check the numbers. If the most units allowed to you make a profit, hire/rent all of them. If not, you probably don't want any.** If there are no limitations, we want as much of the good with IRTS (let's say labor) as possible. This is because the wage is constant, but the MR for another worker is increasing. Eventually the MR of the next worker will be insanely high, higher than the constant MC. However, there will probably be a limit on how many workers we can get. If you can't make a positive profit with as many workers as allowed, less workers will most likely be even worse (since it is the last workers hired that bring in the most revenue). This is assuming constant MC and a constant price.

3) Application

F(L,K)= 5L + 10K   w = 14   r= 26   P = 2   Find L and K in the LR to max profits

See if you can find L^{LR} and K^{LR} by paying close attention to the type of utility function and RTS.

1) **Calculating returns to scale *(solution)***

Compare F(**t**L, **t**K) to **t**F(L,K) (where t>1)

Which is the same as comparing doubling[1] inputs to doubling outputs

What are the returns to scale for the following?

F) $F(L,K) = L^{1/2}K^{1/2}$

$$(tL)^{1/2}(tK)^{1/2} \quad = \quad t(L^{1/2}K^{1/2}) \qquad \textbf{CRTS}$$

If I double inputs, outputs increase by <u>Exactly</u> double. This means I have <u>Constant</u> returns to scale (I doubled my labor force and my capital, and the # of hotdogs produced increased by <u>Exactly</u> double)

G) $F(L,K) = LK$

$$(tL)(tK) \quad > \quad t(LK) \qquad \textbf{IRTS}$$

If I double inputs, outputs increase by <u>more than</u> double. This means I have <u>Increasing</u> returns to scale (I doubled my labor force and my capital, and the # of hotdogs produced increased by <u>More than</u> double)

H) $F(L,K) = \text{Min}\,(2L, K)$

$$\text{Min}\,(2(tL), (tK)) \quad = \quad t(\text{Min}\,(2L, K)) \qquad \textbf{CRTS}$$

If I double inputs, outputs increase by <u>Exactly</u> double. This means I have <u>Constant</u> returns to scale (I doubled my labor force and my capital, and the # of hotdogs produced increased by <u>Exactly</u> double). This assumes we were already at a corner solution

I) $F(L,K) = \text{Min}\,(L^{1/2}, K)$

$$\text{Min}\,((tL)^{1/2}, (tK)) \quad < \quad t(\text{Min}\,(L^{1/2}, K)) \qquad \textbf{DRTS}$$

If I double inputs, outputs increase by <u>less than</u> double. This means I have <u>Decreasing</u> returns to scale (I doubled my labor force and my capital, but the # of hotdogs produced increased by <u>less than</u> double)

J) What is the exponent rule for returns to scale with Cobb-Douglas? (check the slides)

$F(L,K) = L^aK^b$

If $a + b > 1$ IRTS
If $a + b = 1$ CRTS
If $a + b < 1$ DRTS

3) **Application** ***(solution)***

F(L,K)= 5L + 10K   w = 14   r= 26   P = 2   Find L and K in the LR to max profits

Here we have a perfect substitutes problem with CRTS. For perfect substitutes, how much L we want does not affect how much K we want, vice versa. For CRTS, we compare MR to MC to see if which one is greater. Therefore, we will compare P*MPL vs w to see how much labor we want and P*MPK vs r to see how much capital we want.

- For labor

- P*MPL = 10, so our MR from an additional worker is constant at $10.
- w= 14, so our MC for an additional worker is constant at $14.
- We lose 4 dollars for every additional worker we hire, so we hire no one in the LR

- For capital

- P*MPK = 20, so our MR from an additional machine is constant at $20.
- r= 26, so our MC for an additional machine is constant at $26
- We lose 6 dollars for every additional machine we rent, so rent no machines in the LR

$$L^{LR} = 0 \quad K^{LR} = 0$$