SE / IE Answers

1. $u(x,y) = xy \quad P_x = 2 \quad P_y = 2 \quad m = 12 \quad \text{new}: P_y = 3$

- Solve for demand, then Find Points A, B, C

To find demand for A:

- $mrs = \frac{P_x}{P_y} \rightarrow \frac{y}{x} = \frac{P_x}{P_y} \rightarrow y = \frac{P_x x}{P_y}$
- $P_x x + P_y\left(\frac{P_x x}{P_y}\right) = m \rightarrow 2P_x x = m \rightarrow \boxed{x^* = \frac{m}{2P_x} \quad y^* = \frac{m}{2P_y}}$
- $\underline{\underline{x^*}} = \frac{12}{2(2)} = 3 \;/\; \underline{\underline{y^*}} = \frac{12}{2(2)} = 3$
- $u(x,y) = 3 \cdot 3 = \underline{\underline{9}}$

To find demand for B:

• $u(x,y) = xy = 9$ (1) • $x^* = \frac{m}{2P_x} \quad y^* = \frac{m}{2P_y}$ (2) • $P_x = 2 \quad P_y = 3 \quad m = ??$ •

- $(1) \rightarrow (2)$: $u(x,y) = \frac{m}{2P_x} \cdot \frac{m}{2P_y} = 9 \rightarrow \frac{m^2}{2(2) \cdot 2(3)} = 9 \rightarrow m^2 = 216$, $\underline{\underline{m}} = \underline{\underline{14.7}}$
- $\underline{\underline{x^*}} = \frac{14.7}{4} = 3.7 \;/\; \underline{\underline{y^*}} = \frac{14.7}{6} = 2.45$

To find demand for C:

- $\underline{\underline{x^*}} = \frac{12}{2(2)} = \underline{\underline{3}} \;/\; \underline{\underline{y^*}} = \frac{12}{2(3)} = \underline{\underline{2}}$

A

- org Price
- org Income

B

- new price
- Comp Income
- $u(B) = u(A)$

C

- new price
- org Income

	X	Y
A:	3	3
B:	3.7	2.45
C:	3	2

Answer:

	X	Y
(B − A) SE:	.7	−.55
(C − B) IE:	−.7	−.45

2. $u(x,y) = 2x + 3y$ $P_x = 3$ $P_y = 3$ $m = 30$ new: $P_x = 1$

Find Points A, B, and C using Bang per Buck or mrs vs $\frac{P_x}{P_y}$

To Find Point A:

- $\frac{Mu_x}{P_x}$ vs $\frac{Mu_y}{P_y}$ $\rightarrow$ $\frac{\frac{x}{2}}{3} < \frac{\frac{y}{3}}{3}$ $\rightarrow$ Chose All Y
- $x^* = 0$ / $y^* = \frac{m}{P_y} = \frac{30}{3} = 10$
- $u(x,y) = 3(10) = 30$

To Find Point B:

- $u(x,y) = 2x + 3y = 30$ • $P_x = 1$ $P_y = 3$
- $\frac{Mu_x}{P_x}$ vs $\frac{Mu_y}{P_y}$ $\rightarrow$ $\frac{\frac{x}{2}}{1} > \frac{\frac{y}{3}}{3}$ $\rightarrow$ Chose All X
- Chose all X such that $u(B) = u(A) = 30 \rightarrow 2x + \cancel{3y} = 30$
  $x = 15$
- $x^* = 15$ $y^* = 0$

To Find Point C:

- $\frac{Mu_x}{P_x}$ vs $\frac{Mu_y}{P_y}$ $\rightarrow$ $\frac{\frac{x}{2}}{1} > \frac{\frac{y}{3}}{3}$ $\rightarrow$ $x^* = \frac{m}{P_x} = \frac{30}{1}$ $y^* = 0$

Answer:

	X	Y
A	0	10
B	15	0
C	30	0
SE	15	-10
IE	15	0

3. $U(x,y) = \min(2x, 4y)$ $P_x = 2$ $P_y = 4$ $m = 24$ new $P_x = 4$

Find A, B, and C by setting the interior equal

To Find point A:

- $2x = 4y \rightarrow x = 2y$
- $P_x x + P_y y = m \rightarrow P_x(2y) + P_y y = m \rightarrow y(2P_x + P_y) = m$
- $x = 2y \rightarrow x = 2\left(\frac{m}{2P_x + P_y}\right)$

$$\boxed{y^* = \frac{m}{2P_x + P_y} \qquad x^* = \frac{2m}{2P_x + P_y}}$$

- $x^* = \frac{2(24)}{2(2)+4} \rightarrow \frac{48}{8} = 6$ / $y^* = \frac{24}{2(2)+4} = \frac{24}{8} = 3$
- $\underline{\underline{u}} = \underline{\underline{12}}$

To Find Point B

- $U(x,y)$ $\min(2x, 4y) = 12$ (1) • $x^* = \frac{2m}{2P_x + P_y}$ (2) $y^* = \frac{m}{2P_x + P_y}$ • $P_x = 4$ $P_y = 4$ $m = 24$ •

- (2) → (1): $\min\left(2\left(\frac{2m}{2P_x + P_y}\right), 4\left(\frac{m}{2P_x + P_y}\right)\right) = 12 \rightarrow$

$\min\left(\frac{4m}{2(4)+4}, \frac{4m}{2(4)+4}\right) = 12 \rightarrow \min\left(\frac{4m}{12}, \frac{4m}{12}\right) = 12$

$\min\left(\frac{4m}{12}, \frac{4m}{12}\right) = \frac{4m}{12} \rightarrow \frac{4m}{12} = 12 \rightarrow 4m = 144 \rightarrow \underline{\underline{m}} = \underline{\underline{36}}$

- $x^* = \frac{2(36)}{2(4)+4} = \frac{72}{12} = 6$ / $y^* = \frac{36}{12} = 3$

↳ All this work proves a point, but is not necessary. Point A, B are equal for complements. We compensate m just enough to make $u(B) = u(A)$ (so no excess units). Therefore, we consume at the corner. To have $u(B) = u(A)$ then, we consume at the same point.

To find point C:

- $\underline{\underline{x^*}} = \frac{2(24)}{8+4} = \underline{\underline{4}}$ $\underline{\underline{y}} = \frac{24}{12} = \underline{\underline{2}}$

SE is the change in demand due to relative prices. Since you must buy a specific ratio, relative prices are irrelevant. There is no SE for complements.

	x	y
A	6	3
B	6	3
C	4	2
SE	0	0
IE	-2	-1

Answer ↗

4. $u(x,y) = \max(x, 4y)$ $P_x = 3$ $P_y = 4$ $m = 12$ New $P_y = 6$

Solve for A, B, and C by Chosing All x or y.

To Find Point A:

- All x → $x^* = \frac{m}{P_x} = \frac{12}{3} = 4$ / $u(x,y) = \max(4, 0) = 4$
- All y → $y^* = \frac{m}{P_y} = \frac{12}{4} = 3$ / $u(x,y) = \max(0, 4(3)) = 12$
- All y gives more utility, so $x^* = 0$ $y^* = 3$
- $u = 12$

To Find Point B:

- $u(x,y) = \max(x, 4y) = 12$ • $P_x = 3$ $P_y = 6$ $m = ??$
- Compare $\frac{m_u}{P}$ to see which is the better option
- $\frac{mu_x}{P_x}$ vs $\frac{mu_y}{P_y}$ $\quad \underset{x}{\frac{1}{3}} < \underset{y}{\frac{4}{6}}$ → Chose all y
- $4y = 12$, $y^* = 3$ $x^* = 0$

To find Point C:

- All x → $x^* = \frac{m}{P_x} = \frac{12}{3} = 3$ / $u(x,y) = \max(3, 4(0)) = 3$
- All y → $y^* = \frac{m}{P_y} = \frac{12}{6} = 2$ / $u(x,y) = \max(0, 4(2)) = 8$
- Chose All y. $x^* = 0$ $y^* = 2$

Answer:

	X	Y
A	0	3
B	0	3
C	0	2
SE	0	0
IE	0	-1

5. $u(x,y) = 2x + y^{\frac{1}{2}}$ $P_x = 3$ $P_y = 1$ $m = 30$ New: $P_x = 1$

use tangency to solve for demand, then find A, B and C

To Find Point A:

- $mrs = 2/\frac{1}{2}y^{-\frac{1}{2}} \rightarrow 4y^{\frac{1}{2}}$
- $4y^{\frac{1}{2}} = \frac{P_x}{P_y} \rightarrow \boxed{y^* = \frac{P_x^2}{16P_y^2}}$
- $P_x x + P_y\left(\frac{P_x^2}{16P_y^2}\right) = m \rightarrow P_x x = m - \frac{P_x^2}{16P_y} \rightarrow \boxed{x^* = \frac{m}{P_x} - \frac{P_x}{16P_y}}$
- $y^* = \frac{9}{16(1)} = .5625$ / $x^* = 10 - \frac{3}{16} = 9.8125$

$\underline{u(A)} = 20.375$

To find Point B:

- $u(x,y) = 2x + y^{\frac{1}{2}} = 20.375$ • $y^* = \frac{P_x^2}{16P_y^2}$ $x^* = \frac{m}{P_x} - \frac{P_x}{16P_y}$ • $P_y = 1$ $P_x = 1$ $m = ??$
- $y^* = \frac{1^2}{16(1)^2} = .0625$ (don't need m to solve for y, since it is a zero income effect good)
- $u(x,y) = 2x + \left(\frac{1}{16}\right)^{\frac{1}{2}} = 20.375 \rightarrow 2x = 20.125 \rightarrow x^* = 10.0625$

To Find Point C:

- $y^* = \frac{1^2}{16(1)^2} = .0625$
- $x^* = 30 - \frac{1}{16(1)} = 29.9375$

Y, the zero Income effect good *suprise* has no Income Effect. wow!

Answer:

	X	Y
A	9.8125	.5625
B	10.0625	.0625
C	29.9375	.0625
SE	.25	-.5
IE	14.875	0

6. $u(x,y) = x^2 + 5y$ $P_x = 2$ $P_y = 2$ $m = 20$ new: $P_x = 4$

Find demand by comparing utility at All x vs All y

To Find Point A:

- All x $\rightarrow x^* = \frac{m}{P_x} = 10$ / $u(x,y) = 10^2 + 5(0) = 100$
- All y $\rightarrow y^* = \frac{m}{P_y} = 10$ / $u(x,y) = (0)^2 + 5(10) = 50$
- $x^* = 10$ $y^* = 0$ $u(A) = 100$

To Find Point B:

• $u(x,y) = x^2 + 5y = 100$ • $P_x = 4$ $P_y = 2$ $m = ??$ •

- All x $\rightarrow x^2 = 100 \rightarrow x = 10$ / $10 = \frac{m}{P_x} \rightarrow m = 40$
- All y $\rightarrow 5y = 100 \rightarrow y = 20$ / $20 = \frac{m}{P_y} \rightarrow m = 40$
- both All x and All y require the same amount of Income for the same level of utility. Indifferent between x and y.
- $x^* = 10$ $y^* = 0$ or $x^* = 0$ $y^* = 20$

To Find Point C:

- All y $\rightarrow y^* = \frac{m}{P_y} = 10$ / $u(x,y) = (0)^2 + 5(10) = 50$
- All X $\rightarrow x^* = \frac{m}{P_x} = 5$ / $u(x,y) = 5^2 + 5(0) = 25$
- $x^* = 0$ $y^* = 10$

Answer:

	assuming all x for B		All y for B		
	x	y		x	y
A	10	0	A	10	0
B	10	0	B	0	20
C	0	10	C	0	10
SE	0	0	SE	-10	20
IE	-10	10	IE	0	-10