Utility Functions Handout

A big chunk of this class revolves around utility functions (AKA preferences). Bottom line, utility functions tell us how we prefer to consume goods so that we maximize our utility (benefit). Some goods, such as left and right shoes, need to be consumed together in a fixed ratio. Some goods have negative utility, meaning we want none of these goods (spoiled milk). There are many ways to describe preferences, and that is what utility functions do.

Most of the time, our consumption will be limited by some type of budget constraint. Using our utility functions (how we want to consume) and our budget constraint (the restriction on how much we can consume) we can find our demand for each good. **We are going to spend a lot of time finding demand.** Our utility functions include perfect substitutes, perfect complements, Cobb-Douglas, quasilinear concave/convex, and max functions. Throughout this class, we will look at the SAME utility functions from DIFFERENT "angles." First, we look at the utility functions through a basic budget constraint. Later we will look at the utility functions from a different "angle" by using a budget constraint that includes the effects of interest and inflation over time (the intertemporal budget constraint). When we look at utility functions from different angles, their underlying characteristics STAY THE SAME, but solving the problems will be somewhat different. When we are looking at substitutes, regardless of whether we are using the budget constraint or the intertemporal budget constraint, we will want all of the good that gives the most bang for our buck. However, the variables we use to get to the solution are slightly different.

When we go over a new "angle" at which to view the utility functions, lecture will only cover this new angle for some of the functions. This is no reason to get upset at the professors, since it would be impossible to cover every possibility in lecture. You are expected to know how to solve for demand with ALL of the applicable functions from the new angle.

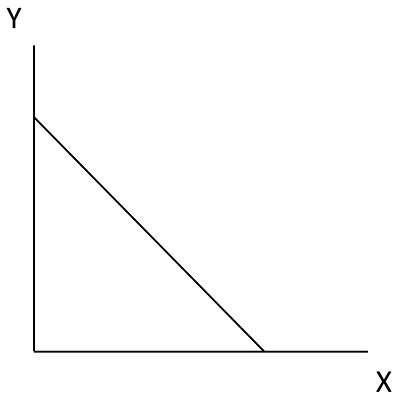
This is a critical thinking class. You should always be thinking about why a problem works the way it does, how it would change if you used a different utility function, and what would happen if you changed the variables. If you start thinking about problems as puzzles, the material is actually quite intuitive, and even fun!

This handout introduces the utility functions using the normal budget constraint (PxX + PyY =m), but the general rules learned here apply to preferences no matter what "angle" we are using. Learning how to identify, characterize, and solve all of the utility functions now will make a significant portion of this class a lot easier. This guide is not exhaustive, nor does it replace lecture, section, hard work, ect. Take your time looking over this, and revisit it while you do the problems provided on the last page.

Best of luck!

Perfect Substitutes

$U(X,Y)= aX +bY$

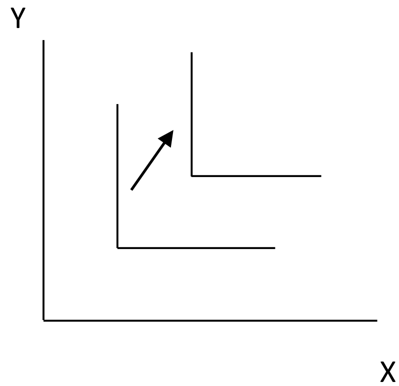


- In real life: Pepsi and coke OR Ice-cream and frozen yogurt OR apples and bananas
- Mathematical form: U(X,Y) = "X + Y" OR "4X + 5Y" OR "10X +Y"
- Any monotonic transformation of "X + Y," such as "$(X + Y)^3$" OR "X + Y +100"
- Since the marginal utility of x and the marginal utility of y (Mux and Muy) are constant, the marginal rate of substitution (MRS) is constant (EX MRS= -2 OR MRS = 5)
- Since the slope of the Indifference curves (IDCs) is the MRS, the IDCs are represented by a constant slope
  - **Solving**
- <u>Key characteristic (Conceptual):</u> **Buy all of the good that gives the most BANG for your buck (utility per dollar spent)**
- If both goods give the same bang for your buck, then you are indifferent between the two goods. You could consume all of good X, all of good Y, or some combination of the goods. Because the slope of the BC is the same as the slope of the IDC, any bundle on the BC are also on the same IDC (meaning all affordable points have the same utility)
  - **Methods** (know all of the different methods!)

1. Sketch a graph: Compare the MRS (Mux/Muy) to the price ratio (Px/Py)
2. Compare Bang for Buck: Compare the utility per dollar for both goods (Mux/Px and Muy/Py)
3. Compare utility levels: find utility if all good 1 is purchased versus all good 2

✓ CHECK: AM I GETTING THE MOST UTILITY POSSIBLE? DO I SPEND ALL OF MY INCOME?

Perfect Complements (Leontief) U(X,Y)= Min(aX,bY)



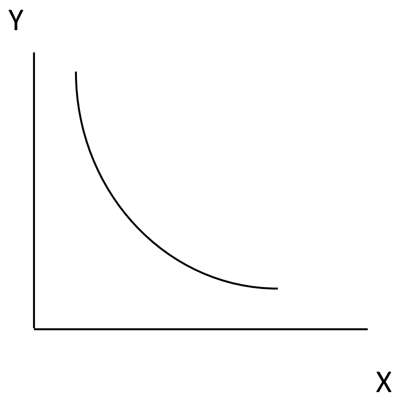
- In real life: Left and right shoes OR tea and sugar
- Mathematical form: U(X,Y)= "Min (X, Y/2)" OR "5Min (X, Y)" OR "Min (5X, 5Y)"
- Any Monotonic transformation of "Min (aX, bY)" such as "Min (aX, bY) + 100" OR "3Min (aX, bY)" (Note: monotonic transformations do NOT change the ratio at which you want to consume perfect complements. They do change the total and marginal utility).
    - **Solving**
- Key Characteristic (Conceptual): **Consume in a fixed ratio, where the two variables inside the Min function are equal.** AKA, set the interior equal
- I want to make the MOST bundles I can, while paying as little as possible. This means that, **if possible, I will not buy excess of one good that does not contribute to a bundle**
- The Min function outputs the lower of 2 numbers. If our utility function is U(X, Y)= Min (X, Y) and we consume at (10, 50), our function is Min (10, 50), which outputs 10. If we consume at (10, 10), our utility is once again 10, the lowest number
- The excess of Y at (10, 50) increases the amount we must pay, but not our utility
- If U(X, Y) = Min (X, Y) and X is fixed at 15, how much Y is optimal? Do large Y values help?
    - **Example: Peanut (B)utter and (J)elly** U(B,J)= Min(4B, 2J)
- I want to make the most sandwiches I can.

1. Set the interior equal to find the fixed ratio at which to consume
    a. Consume at a fixed ratio 4B = 2J → B = J/2. This means that for every unit of peanut butter, we want 2 units of jelly. This is the ratio needed to make a sandwich
2. Plug B=J/2 into the budget constraint
3. Once one variable is solved, plug it into the ratio (from setting the interior of the Min function equal) to get the other variable

✓ CHECK: DO I HAVE AN EXCESS OF EITHER GOOD? DO I SPEND ALL OF MY INCOME?

Cobb-Douglas

$U(X,Y)= aX^{\alpha}bY^{\beta}$



- Mathematical form: $U(X,Y)=$ “XY” OR “$X^{1/2}5Y^{1/3}$” OR “$2X^2Y^3 + 100$”
- Any monotonic transformation of “$aX^{\alpha}bY^{\beta}$”

  - **Solving**

- Key Characteristic (Mathematical): **Use Tangency, which is the marginal rate of substitution set equal to the price ratio (MRS = Px/Py)**
- MRS is always diminishing (our graph is convex), allowing us to use the tangency condition. The Tangency condition is *only* possible with DMRS. The slope of the budget line is Px/Py and the slope of the IDC is the MRS, so tangency is where MRS=Px/Py
- Tangency tells us where are budget line (what we can afford) intersects our IDC (what we want). Tangency finds the highest IDC that our budget constraint intersects, AKA the best point we can afford.
- Cobb-Douglas exponent trick: The fraction of the exponents tells us the proportion of income we spend on each good (Ex if $U(X,Y)= X^{1/2}Y^{1/2}$ spend half of your income on each)

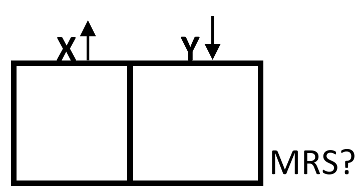
  - **Steps**

1. Find MRS and test for diminishing MRS (DMRS).[1] Set MRS=Px/Py
2. Plug answer from 2 (solved for either X or Y) into the budget constraint
3. Plug in solved variable into the budget constraint or MRS=Px/Py to find the other variable

✓ Check: is my math right? Understanding partial derivatives is crucial for Cobb-Douglas

Test for DMRS

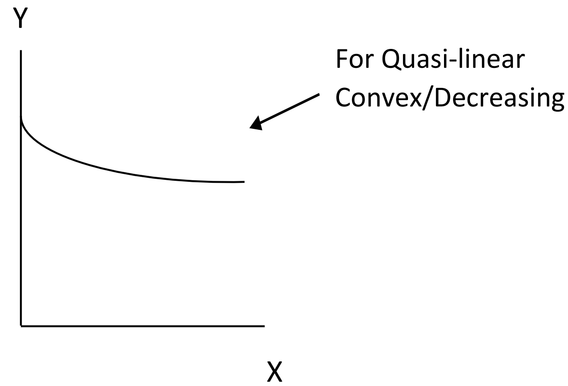
If MRS decreases as X increases and Y decreases, the function has DMRS. If MRS is constant for an increase in X, but is decreasing for a decrease in Y, it is still has DMRS. When one is constant, the other dominates.

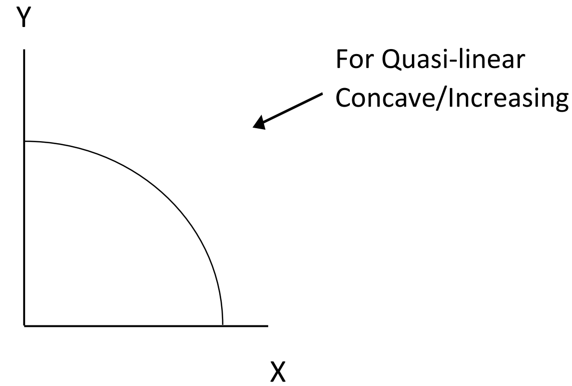


[1] Cobb-Douglas always has DMRS. We test for to ensure we have a Cobb-Douglas function, and because it is a good step to show on tests.

Quasi-Linear

U(X,Y)= V(X) + bY





- Examples: "X + ln(Y)" OR "2X + $Y^{1/2}$" OR "X^2 + Y"
- Any monotonic transformation of V(X) + bY

    - **Quasi-linear Convex/Decreasing**

- In Real life: cooking pans and ingredients OR pencils and all other goods
- In mathematical form: U(X,Y)= "ln(X) + Y" OR "X + $2Y^{1/2}$"
- Diminishing MRS, the function is convex

    - **Example: cooking (P)ans and (I)ngredients** $U(P,I) = 2P^{1/2} + I$

- Notice that Mu of ingredients is constant, while the Mu of pans is diminishing. This means that at some point buying an extra unit of ingredients will increase your utility by more than an extra pan. This is because, at some point, having more pans does not help you very much.
- The demand for the diminishing Mu good does not include income (M) in the equation (Ex $P^*=P_p^2/P_I^2$). We call this a zero income-effect good, since the level of income does not change the amount demanded. In this case, pans are a zero income-effect good. For example, you want 1 pan regardless of how much money you have, and you will not buy any ingredients until you have one pan.

    - **Solving**

- <u>Key Characteristic (Mathematical):</u> **Use Tangency. Do not consume any of the good with constant Mu until you have consumed the demanded amount of the good with diminishing Mu (the zero-income effect good).**

1. Find MRS and test for IMRS or DMRS. Decreasing quasi-linear has DMRS, so use tangency
2. MRS=Px/Py (you will solve for the variable with diminishing Mu in this step)
3. Plug the solved variable into the budget constraint

✓ CHECK: IF I DON'T HAVE ENOUGH INCOME, DID I MAKE SURE TO GET AS MUCH OF THE DIMINISHING GOOD AS POSSIBLE?

- **Quasi-Linear Concave/Increasing**

- In Real Life: $10 dollar bills and cocaine
- Mathematical form: U(X,Y) = "$X^2 + 2y$" OR "$X + 2Y^3$"
- The function does not have DMRS and is therefore *not* convex. You *cannot* use tangency. The function has increasing MRS, meaning the graph is concave.

  - **Example $10 (B)ills and and (C)ocaine** U(C,O)= $2C + O^2$

- $10 has constant Mu, while Cocaine has increasing Mu. Eventually the good with the increasing Mu will be better than the good with constant Mu. The first few $10 bills have higher Mu than small amounts of cocaine, but since cocaine is addictive, larger amounts of cocaine will have higher MU the $10 dollars.
- If you could only consume 1 unit total, would you want cocaine or $10? What If you could consume 100 units total? This is just an example, **don't do drugs!**

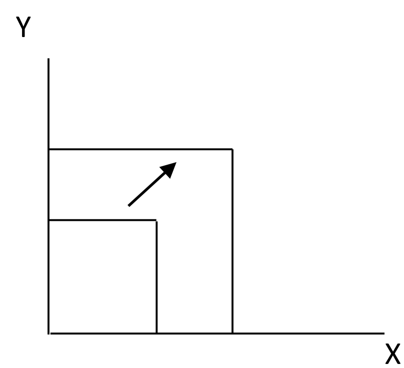
  - **Solving**

- <u>Key Characteristic (conceptual):</u> consume all of the good that gives the most utility

1. Find MRS, test to find DMRS or IMRS. Quasi-Linear Concave has IMRS
2. IMRS, can't use tangency, so consume all of one or the other
3. Compare all X vs all Y and consume all of the good that gives the most utility

Max

U(X,Y)= Max(X,Y)



- Examples "Max (3X, Y)" OR "6Max(X, 3Y)" OR "Max(2X, 2Y) +100"
- Any monotonic transformation of "Max(X,Y)"
- Max outputs the largest of 2 numbers: U(X,Y)=Max(100,0) = 100 AND Max(100,50)=100
- Our utility in both cases is 100, but the first is cheaper, so consume all of one good.
- Notice how the cheapest points on each IDC are the corners (all X or all Y)

  - **Solving and Methods**

- <u>Key Characteristic (conceptual):</u> consume all of the good that gives the most utility

1. Compare utility levels: compare all X vs all Y and get the good with the most utility

Demand Problems

Perfect Substitutes

1. $U(X, Y) = 5X + 2Y$ Px=6 Py=2 M=24 What is X* and Y*?
2. $U(X, Y) = 2X + 3Y$ Px=2 Py=3 M=12 What is X* and Y*?
3. $U(X, Y) = 3X + Y$ No prices/income given[2] What is X* and Y*?

Perfect Complements

1. $U(X, Y) = \text{Min}(2X, Y)$ Px=10 Py=2 M=28 What is X* and Y*?
    a. What is Mux at (1,2)? At (3,2)? At (1,4)?
2. $U(X, Y) = 5\text{Min}(X, Y/5)$ Px=5 Py=1 M=30 Y=20[3] What is X*?
    a. What would X* and Y* be if Y was not fixed at 20?

Cobb-Douglas

1. $U(X, Y) = X^2Y$ No prices/income given What is X* and Y*?
    a. What portion of income do you spend on X and Y if $U(X, Y) = x^2y$? If utility = $x^{1/2}y$?

Quasi-Linear Convex (Decreasing)

1. $U(X,Y) = 2X^{1/2} + Y$ No prices/income given What is X* and Y*?
    a. What would X* and Y* be if M=30 Py=5 Px=1
    b. What would X* and Y* be if M=3 Py=5 Px=1
2. $U(X, Y) = X + \ln(Y)$ No prices/income given What is X* and Y*?

Quasi-Linear Concave (Increasing)

1. $U(X, Y) = X^2 + 5Y$ Px=5 Py=3 M=30 What is X* and Y*?

Max

1. $U(X, Y) = 2\text{Max}(3X, Y)$ Px=4 Py=1 M=20 What is X* and Y*?

---

[2] When you are not given prices with a substitutes problem, you will need to find *conditional* demand. Conditional demand is an "if then" statement, which tells you what the demand will be "if" the prices are a certain amount.

[3] Assume you are forced to *purchase* 20 units of Y