Meeting 6A. Worksheet + Solutions.
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Problem 1

Derive the intertemporal budget constraint without inflation, then with inflation.

Problem 2

Assume c_1 is a normal good. No inflation. The interest rate increases. You are a lender. Draw the SE, IE on a graph.

Problem 3

Derive the budget constraint for the insurance model.
What is the price of consumption in the bad state?
Draw the budget constraint on a graph.

Problem 4

No income in period 1. Income in period 2 is \$1100.
$\pi = 10\%$. $r = 10\%$. Price of consumption in period 1 is 1.
$U(c_1, c_2) = c_1^2 c_2$.
What levels of consumption will Bob choose for each period?

Meeting 6. Problem Set 6.

Solutions.

Problem 1

Without inflation. Suppose you are a lender.

Then $c_1 < m_1$. So you put $m_1 - c_1$ in savings in period 1,

which becomes $(1+r)(m_1 - c_1)$ in period 2.

So in period 2: $c_2 = (1+r)(m_1 - c_1) + m_2$

so $(1+r)c_1 + c_2 = (1+r)m_1 + m_2$ ← in future values

so $c_1 + \frac{1}{1+r} c_2 = m_1 + \frac{1}{1+r} m_2$ ← in present values

With inflation. Let m_2 be in terms of future consumption (in real terms).

Let p_2 be the money price of future consumption.

cost of consumption in period 2 $= p_2 \cdot c_2$

money available in period 2 $= (1+r)(m_1 - c_1) + p_2 \cdot m_2$

so the budget constraint is:

$$p_2 c_2 = (1+r)(m_1 - c_1) + p_2 \tilde{m}_2$$

so $c_2 = \frac{1+r}{p_2} \cdot (m_1 - c_1) + \tilde{m}_2$, and $p_2 = 1+\pi$

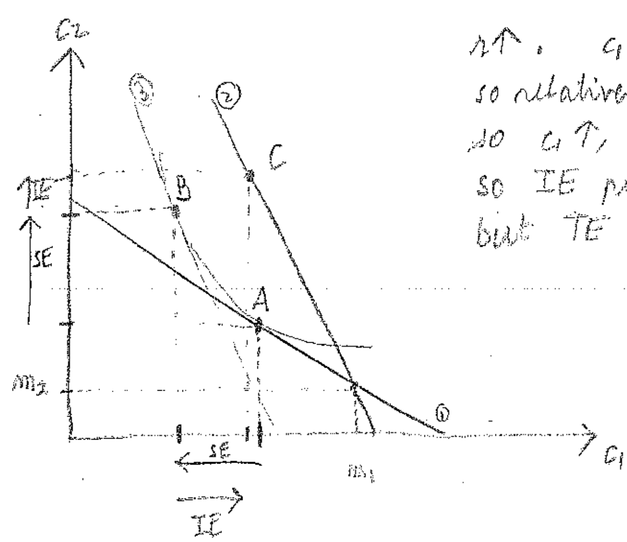
so $c_2 = \frac{1+r}{1+\pi}(m_1 - c_1) + \tilde{m}_2$, let $1+\rho = \frac{1+r}{1+\pi}$

so $c_2 = (1+\rho)(m_1 - c_1) + \tilde{m}_2$

so $(1+\rho)c_1 + c_2 = (1+\rho)m_1 + \tilde{m}_2$

so $c_1 + \frac{c_2}{1+\rho} = m_1 + \frac{\tilde{m}_2}{1+\rho}$, (with $\tilde{m}_2 = \frac{\text{income in period 2}}{1+\pi}$)

Problem 2



$r\uparrow$. c_1 normal good.

so relative income increases.

so $c_1\uparrow$, $c_2\uparrow$.

so IE positive.

but TE unknown.

Problem 3

Good state: $c_g = m - \gamma K$

Bad state: $c_b = m - L - \gamma K + K = m - L + (1-\gamma) K$

so $\quad c_b - (m - L) = (1-\gamma) K$

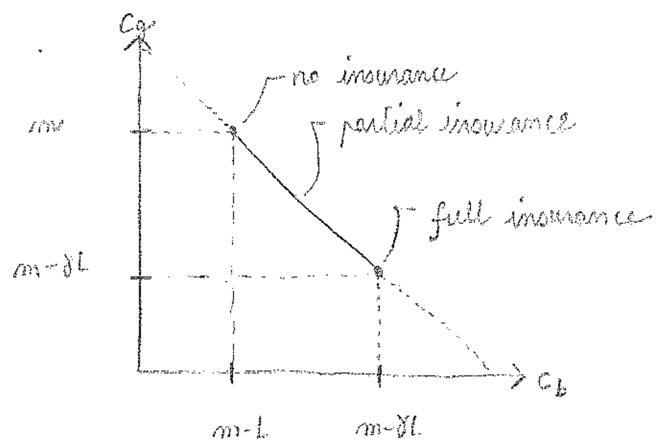
so $\quad K = \dfrac{c_b - (m-L)}{(1-\gamma)}$. let's plug that into c_g.

so $\quad c_g = m - \gamma \cdot \dfrac{c_b - (m-L)}{(1-\gamma)}$

so $\quad c_g = m - \dfrac{\gamma}{1-\gamma} \cdot c_b + \dfrac{\gamma}{1-\gamma} \cdot (m - L)$

so $\quad \dfrac{\gamma}{1-\gamma} c_b + c_g = m + \dfrac{\gamma}{1-\gamma} \cdot (m - L)$

so $\dfrac{\gamma}{1-\gamma}$ is the price of consumption in the bad state.



Problem 4

Let's derive the IDC. (see Problem 1).

We get $\quad c_1 + \dfrac{c_2}{1+e} = m_1 + \dfrac{\tilde{m}_2}{1+e}$

and $\quad 1+e = \dfrac{1+\pi}{1+\pi} = 1 \quad$ and $\quad m_1 = 0, \ \tilde{m}_2 = \dfrac{1100}{1+\pi} = 1000$

so we get $\quad c_1 + c_2 = 1000$.

and $\quad u(c_1, c_2) = c_1^2 c_2 \quad$ so $\quad MRS = \dfrac{2c_1c_2}{c_1^2} = -\dfrac{2c_2}{c_1} = -\dfrac{1}{1}$ (tangency condition)

So $\quad 2c_2 = c_1$.

so $\quad 3c_2 = 1000 \quad$ so $\quad c_2 = \dfrac{1000}{3} \quad$ and so $\quad c_1 = \dfrac{2000}{3}$.